A Computer Simulation for Quantal Interference

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Abstract

Interference of particles is one of the central phenomena of quantum mechanics. The computer program *InterferenceSimulator* demonstrates two-slit Fresnel interference patterns with one, the other, or both slits open. A magnetic flux situated between the two slits allows demonstration of the Aharonov-Bohm effect. Simulations with short de Broglie wavelengths illustrate the classical limit of quantum mechanics. Because of the universality of wave phenomena, this program also demonstrates the geometrical-optics limit of wave optics for small wavelengths.

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I. THE PURPOSE

The iconic introductions to quantum mechanics by Richard Feynman emphasize interference as the "mysterious behavior ... [at] the heart of quantum mechanics"[1] and claim[2] that "Any other situation in quantum mechanics, it turns out, can always be explained by saying 'You remember the case of the experiment with the two holes? It's the same thing.' "[3] This central mystery has been the subject of numerous direct experimental tests.[4–7]

The Feynman treatments are qualitative, not quantitative, and the experimental tests, while impressive in the extreme, are too elaborate to be reproduced in a typical undergraduate laboratory. This paper introduces the computer program *InterferenceSimulator* that readily and rapidly simulates two-slit particle interference experiments — with one slit open, with the other slit open, or with both slits open — under a wide variety of experimental conditions. With this program it is easy to demonstrate destructive interference and constructive interference. It is easy to show the classical limit of quantum mechanics by making the slits wide compared to the de Broglie wavelength.

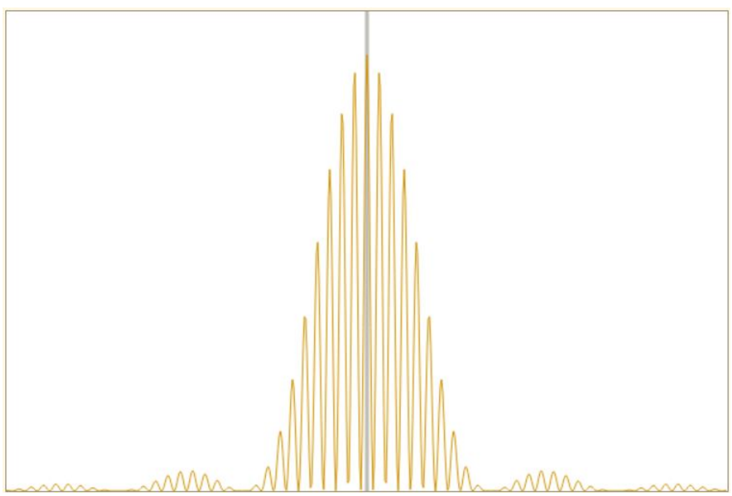

Figure 1: The default display of *InterferenceSimulator*.

Program *InterferenceSimulator* also simulates the Aharonov-Bohm effect,[8–10] wherein the presence of a magnetic field within the barrier between the two slits affects the interference pattern, despite the fact that the particle is rigorously excluded from that barrier! The simulation makes plain the quantitative character of the effect, which has been much misrepresented. For example, comparison of figures 15-5 and 15-7 in volume II of the Feynman Lectures[11] suggests incorrectly that the interference pattern slides back and forth rigidly (without changing shape) as the magnetic field changes, whereas in fact the interference pattern wiggles within a field-independent envelope.

While the primary role of *InterferenceSimulator* is to demonstrate quantal interference effects, the phenomenon of interference is universal among waves, so the simulation necessarily demonstrates interference in optical or acoustic waves as well. In this role it is particularly valuable for showing the geometrical-optics limit of wave optics in the limit of small wavelengths.

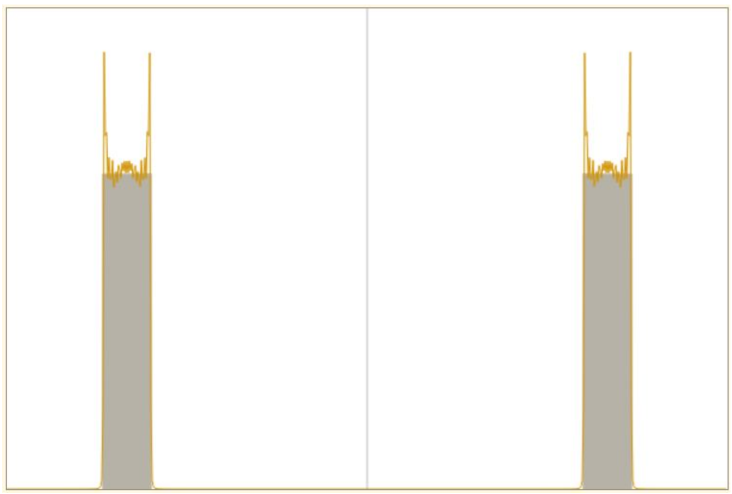

Figure 2: The display of *InterferenceSimulator* in a short-wavelength situation, demonstrating the classical limit of quantum mechanics (or the geometrical-optics limit of wave optics). The gray boxes show the "ray-optics spotlights" that would be produced if particles behaved classically.

II. THE MODEL SIMULATED

The program simulates Fresnel rather than Fraunhofer diffraction, because only in the Fresnel case does a classical limit exist.

A point source a distance $R_s + R_o$ from the observation plane emits monochromatic de Broglie waves

$$\psi(\vec{r}) = \frac{A}{r} e^{i(kr-\omega t)}, \tag{1}$$

that pass through the two infinitely tall slits of width w separated by a distance d,

$$w < d. \tag{2}$$

Completely enclosed within the center-post between the two slits is a static magnetic field with flux Φ. (Positive flux corresponds to magnetic field out of the page.) If the interfering particle possess charge q (the simulation uses particles with the charge of the proton), define the phase factor

$$\phi = \frac{q}{\hbar c}\Phi. \tag{3}$$

(This equation uses Gaussian units. To convert to SI, replace "c" with "1".) The simulation uses the short wavelength (Kirchhoff) approximation

$$\lambda = \frac{2\pi}{k} \ll R_s, R_o \tag{4}$$

and the paraxial approximation

$$d, x \ll R_s, R_o. \tag{5}$$

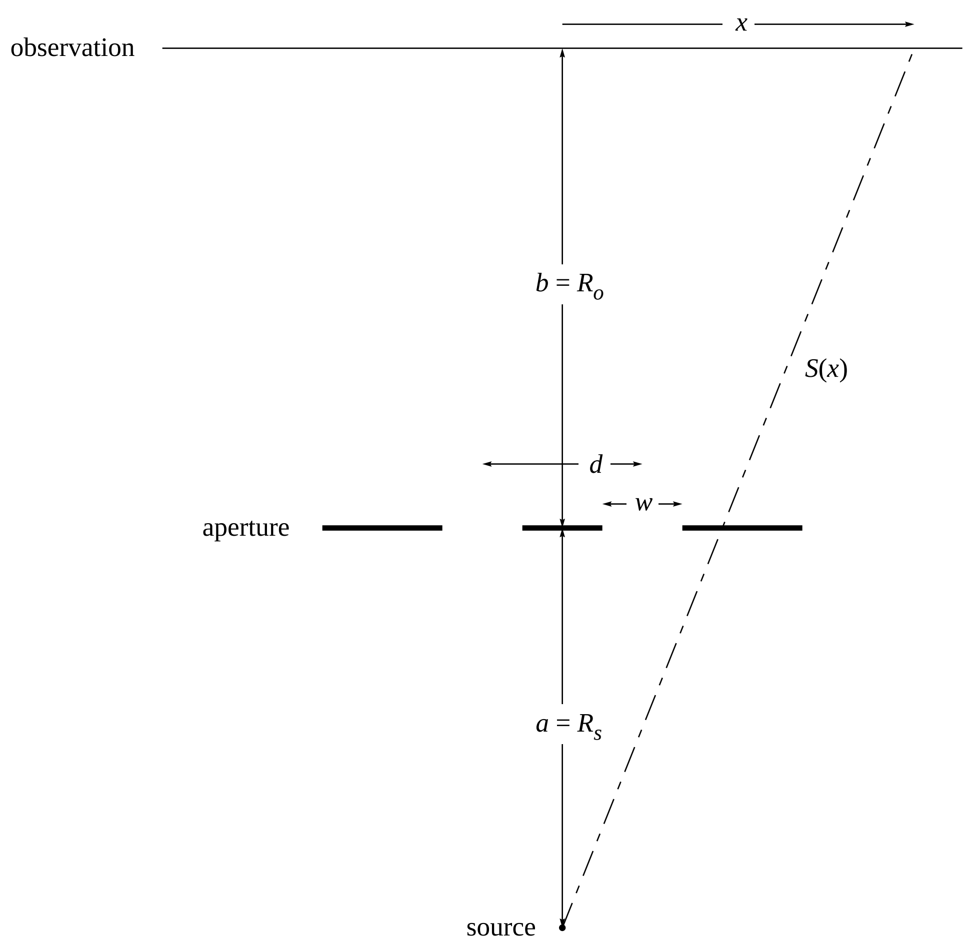

Figure 3: The geometry of the two-slit interference experiment.

Classical wave theory[12] and quantum mechanics[9,13] agree on the answer to this problem: the wavefunction at x due to the right slit is

$$\begin{aligned}\psi_R(x) &= \frac{A}{\sqrt{2i}}\frac{e^{i(kS(x)-\omega t)}}{R_s+R_o}\int_{V_1}^{V_2} e^{i(\pi/2)V^2}\,dV \\ &= \frac{A}{\sqrt{2i}}\frac{e^{i(kS(x)-\omega t)}}{R_s+R_o}\left\{[C(V_2)-C(V_1)]+i[S(V_2)-S(V_1)]\right\}. \end{aligned} \tag{6}$$

where $C(V)$ and $S(V)$ are the Fresnel integrals[14] and where

$$V_2 = \left[\frac{2}{\lambda}\left(\frac{1}{R_s}+\frac{1}{R_o}\right)\right]^{1/2}\left(\frac{R_s}{R_s+R_o}x-\tfrac{1}{2}d+\tfrac{1}{2}w\right) \tag{7}$$

$$V_1 = \left[\frac{2}{\lambda}\left(\frac{1}{R_s}+\frac{1}{R_o}\right)\right]^{1/2}\left(\frac{R_s}{R_s+R_o}x-\tfrac{1}{2}d-\tfrac{1}{2}w\right). \tag{8}$$

The wavefunction $\psi_L(x)$ at x due to the left slit is the same, except that every "d" is replaced by "$-d$". Reflection symmetry requires that $\psi_R(-x)=\psi_L(x)$, and it is easy to show that these expressions adhere to that requirement.

The wavefunction at x due both slits is[9] (up to an overall phase factor)

$$\psi_L(x)+e^{i\phi}\psi_R(x) \tag{9}$$

so the resulting probability density is

$$\begin{aligned}&|\psi_L(x)|^2+|\psi_R(x)|^2+2\,\Re e\{e^{i\phi}\psi_L^*(x)\psi_R(x)\} \\ &= |\psi_L(x)|^2+|\psi_R(x)|^2+2\cos\phi\,\Re e\{\psi_L^*(x)\psi_R(x)\}-2\sin\phi\,\Im m\{\psi_L^*(x)\psi_R(x)\}. \end{aligned} \tag{10}$$

For $\Phi=0$ this probability density is symmetric; for $\Phi\neq 0$ it is in general asymmetric, but it oscillates between the two symmetric and flux-independent envelopes of

$$|\psi_L(x)|^2+|\psi_R(x)|^2\pm 2|\psi_L(x)||\psi_R(x)| = (|\psi_L(x)|\pm|\psi_R(x)|)^2. \tag{11}$$

III. USES

The easiest way to casually use *InterferenceSimulator* is to visit

`http://www.oberlin.edu/physics/dstyer/InterferenceSimulator`.

The program's controls and output are self-explanatory. Those wishing to probe in more detail will find the JavaScript source code freely available through

`http://sourceforge.net/projects/interferencesimulator/`.

It is released to the public without warranty under the terms of the GNU General Public License, version 3.

The most straightforward use of *InterferenceSimulator* is to show the interference pattern resulting from one slit, then from the other, and finally from both. It will be obvious that at some points the probability from both slits is *more than* the sum of the probabilities from each slit, and equally obvious that at other points the probability from both slits is *less than* the sum — sometimes it is even zero! One can then make the wavelength short to demonstrate the classical limit of quantum mechanics — and in this limit, to high accuracy, the third pattern *is* the sum of the first two.

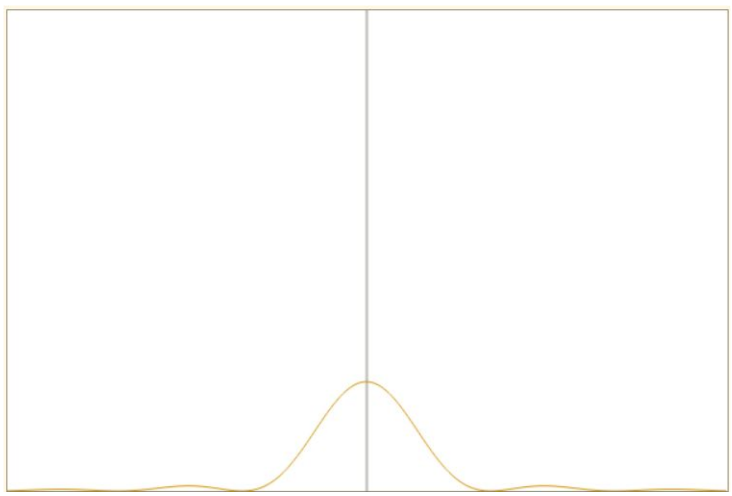

Figure 4: The default condition of *InterferenceSimulator*, as in figure 1, except that only the left slit is open.

When demonstrating the Aharonov-Bohm effect, a good strategy is to first show that the magnetic flux has no affect on the interference pattern when the right slit alone is open, and similarly for the left. But the flux *does* affect the interference pattern when both slits are open.

When both slits are open, changing the magnetic flux slider results in the interference pattern creeping to the right or left between the two envelopes given in equation (11). This surprising (and, may I add, beautiful) effect cannot be fully appreciated through a static image, but figure 5 at least shows that for non-zero flux the interference pattern need not be symmetric. (The flux-independent envelopes, however, *are* always symmetric.)

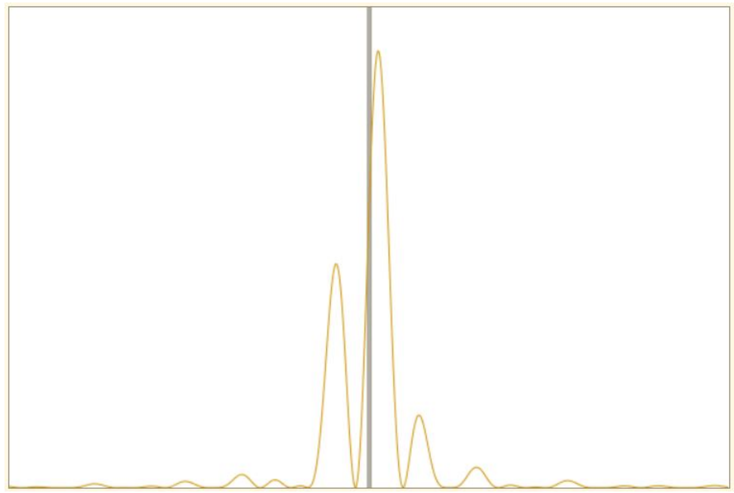

Figure 5: When there is a magnetic flux between the two slits (Aharonov-Bohm effect), the interference pattern is not necessarily symmetric. The pattern in this figure was generated under default conditions, except that $\Phi = 8.7 \times 10^{-16}$ Wb, slit width is 2.4 μm, and momentum is 4.2×10^{-29} kg·m/s.

InterferenceSimulator has been used to good effect in introducing quantum mechanics both to physics students and to a general audience. Experimental results that previously seemed hard to grasp were rendered immediate and crisp. Of course, the interpretation of these results remains counterintuitive!

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