Philosophy of my first-semester junior-level quantum mechanics course

Dan Styer; 27 June 2002, modified 9 July 2002

This course weaves together experiments, reasoning, formalism, and applications into a strong network of understanding surrounding the four pillars of quantum mechanics: quantization, probability, interference, and entanglement.

These ideas are developed first in a "two basis state" system called "spin-$\frac{1}{2}$," and then extended to the case of continuum wave mechanics. I take this approach because:

- Two-state systems are mathematically simpler than wave mechanics situations, allowing us to concentrate on physics rather than mathematics.
- The best experimental tests of quantum mechanics have in fact been performed on two-state systems.
- Students don't have an emotional commitment to spin-$\frac{1}{2}$, and so have fewer ingrained preconceptions. (Those students who, on the grounds of nostalgia, refuse to accept that "an atom with a well-defined momentum has no position" are quite willing to open their minds to the possibility that "an atom with $s_z = +\hbar/2$ has no value for s_x".)
- Two-state systems are, in some sense, the most starkly quantal of all systems. It is in these systems that the pillars of quantum mechanics express themselves most unambiguously. Students cannot delude themselves into thinking that their classical ideas are "nearly correct."
- The most exciting contemporary application of quantum mechanics, quantal computation, is built upon two-state systems.

Note: Photon polarization represents an alternate realization of a two-state system. I prefer the spin-$\frac{1}{2}$ atom because:

- The photon is inherently relativistic.
- Most students (correctly!) don't visualize a photon as a localized discrete entity, as they do an atom.
- It is easy to give a correct microscopic picture for the workings of a Stern-Gerlach analyzer, but it is difficult to produce a microscopic picture for the workings of a polarization analyzer. (Consider, for example, a crystal of birefringent calcite: It's not easy to give a macroscopic picture showing why the speed of light should depend on direction and on polarization state — but a microscopic picture, treating light as photons being absorbed and reemitted, is harder still.)
- A full satisfactory treatment of the photon is quite elaborate, involving, for example, electric and magnetic field operators as well as photon number operators. The photon itself relates to an eigenstate of the energy operator, which in turn is an integral over all space of the squares of the **E** and **B** field operators. For this reason, it is impossible at the undergraduate level to give a legitimate treatment

of the classical limit for a photon. (I tell students: "There is always a tendency to view the photon as a little bundle of electric and magnetic fields, a "wave packet" made up of these familiar vectors. This view is completely incorrect. In quantum electrodynamics, in fact, the electric field is a classical macroscopic quantity that takes on meaning only when a large number of photons are present.")

The wave mechanics portion of the course emphasizes:

- Time development.
- The momentum representation as an equal partner to the position (i.e. configuration space) representation.

Course themes

In this course, emphasis is upon actions (e.g. "an atom moves from $\mathbf{x}_1$ to $\mathbf{x}_2$") and their associated amplitudes, and only secondarily upon the more abstract concept of quantal state ($|\psi\rangle$ or $\psi(\mathbf{x})$ or $\tilde{\psi}(\mathbf{p})$).

The state vector (or wavefunction) is an algorithmic tool for calculating probabilities, not a physical entity. For example, the wavefunction exists in configuration space, not physical space. (For a two-particle system, there is one wavefunction in 6-dimensional configuration space, not two wavefunctions in 3-dimensional physical space.) The "collapse of the wavefunction" is not a physical process — it is no more worrisome than is throwing out scratch paper.

Powerful mathematical tools (such as Dirac notation, Fourier transforms, and computer simulation) are used throughout. But these tools do not take center stage. The connection between mathematics and physics is carefully maintained, and each is used to illuminate the other. For example, the completeness relation,

$$1 = \sum_n |n\rangle\langle n|,$$

means that an interferometer reconstructs the incoming quantal state.

No visualization or analogy for quantum mechanics is perfectly apt. Nevertheless, as we study quantum mechanics such metaphors form in our minds spontaneously. It is best to discuss these metaphors openly so that we may hold our imperfect metaphors critically rather than naively.

No single flat map accurately represents our spherical world. Yet a collection of flat maps — an atlas — can be stitched together to give a reasonably accurate picture of world geography. In order to do this stitching, one must understand the limitation of each flat map. Similarly, no single metaphor encompasses all of quantum mechanics, yet a collection of metaphors (each inaccurate, but each with its limitations understood) can be stitched together to give a reasonably accurate picture of quantum mechanics.

Course aphorisms

"In two slit interference, quantum mechanics cannot determine which slit the electron went through." This statement reflects not the poverty of quantum mechanics, but its richness. In classical mechanics, an electron must have a position — it must pass through one slit or the other. In quantum mechanics an electron might have a position, but there is an infinitely rich variety of other possibilities as well.

It is no failure of our instruments that they cannot measure what does not exist.

The English language was invented by people who didn't understand quantum mechanics, so it is unsurprising that the language lacks a concise, accurate description of many quantal phenomena.

The strangeness of quantum mechanics carries us to the brink of implausibility — *but not beyond.*