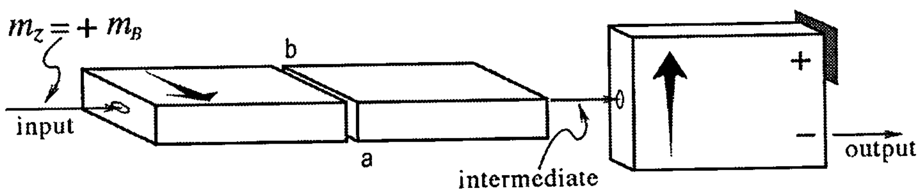
10

Amplitudes

10.1 The amplitude framework



Recall from the first three quantal interference experiments (pages 65–67) that in the above apparatus, the probability of passing from the initial state (at input with $m_z = +m_B$) to the final state (at output with $m_z = -m_B$) is

situation	probability
branch a open	1/4
branch b open	1/4
both branches open	0

Clearly the probability of passing through both branches does not equal the sum of the probability of passing through branch a plus the probability of passing through branch b. On the other hand, it seems natural to ascribe the total probability to some sort of an "influence through branch a" plus an "influence through branch b". (Recall that optical interference was described by a similar picture, where the "influence through a slit" was either the undulation due to that slit or the stopwatch hand associated with a photon passing through that slit.) It somehow seems unscientific to call these things "influences", a word beloved by mediums and witches, so they are called "amplitudes" (or sometimes "probability amplitudes"). At the moment the existence of amplitudes is nothing but a reasonable surmise, but this guess will turn out to be an excellent one, supported by reams of evidence (to be reviewed later in this chapter). For now, however,

our task is to firm up the concept of amplitudes, and, in particular, to find a mathematical representation for them.

The salient feature of amplitudes is that the sum of an "amplitude to pass through branch a" plus an "amplitude to pass through branch b" can lead to a total probability of zero. Thus an amplitude cannot be represented by an intrinsically positive number, because two positive numbers cannot add up to zero. There are, however, many classes of mathematical entities for which two elements of the class *can* add to zero. One such class is the real numbers, as demonstrated by $(+0.7)+(-0.7) = 0$. We will see in section 11.1 that the class of real numbers cannot adequately represent all possible amplitudes. Instead, amplitudes must be represented by two-dimensional arrows* similar to the rotating stopwatch hands of the optical interference experiment (section 8.4). If there are several ways of going from the initial to the final state, then the "total amplitude" for doing so is just the sum of the several individual amplitudes, where arrows are summed by placing them tail to head as described on page 61. The probability of going from the initial to the final state is just the square of the magnitude of the total amplitude arrow.

Let us see how the general ideas of amplitudes and probabilities presented above can explain the first three quantal interference experiments from the preceding chapter. The amplitude to go from input to output via branch a is represented by an arrow of magnitude $\frac{1}{2}$ pointing right: $\rightarrow$. The amplitude to go from input to output via branch b is represented by an arrow of magnitude $\frac{1}{2}$ pointing left: $\leftarrow$. When both branches are open, the total amplitude is represented by the sum of the two arrows, which is just an arrow of magnitude zero.

situation	sum of amplitudes	probability
branch a open	$\rightarrow$	1/4
branch b open	$\leftarrow$	1/4
both branches open	·	0

Now we can firm up the vague phrase "the atom goes through both branches" introduced in the last chapter. Its precise meaning is simply that there is an amplitude for the atom to go through either branch.

> *Technical aside:* The above paragraph illustrates important general techniques for assigning amplitude arrows. The magnitude of an arrow can be fixed by knowing the corresponding probability, because the magnitude is just the square root of the prob-

* These amplitude arrows are not related to the magnetic needle arrows introduced in chapter 2. This book represents magnetic needles by arrows with filled arrowheads and amplitudes by arrows with open arrowheads.

ability. (In the situation above, $\frac{1}{2} = \sqrt{\frac{1}{4}}$.) The angles between the arrows are harder to find: they must be uncovered through the results of interference experiments. Section 11.1 (page 86) works out such an assignment problem in some detail.

Amplitude arrows are mathematical tools that permit the computation of probabilities, they are not physical entities that are actually located in space and observable if only you were to look hard enough.[†] You must not think that there are two real live physical arrows out there, one flying through branch a and the other flying through branch b. For one thing, the amplitude arrows are dimensionless — an arrow is not $\frac{1}{2}$ inch long or $\frac{1}{2}$ millimeter long, it is just $\frac{1}{2}$ long. For another, the orientation of the arrows is not specified exactly. If each arrow in any given problem is rotated by the same angle, then the same probabilities will result. The association between the physical entity (an amplitude) and its mathematical representation (an arrow) is not unique.[‡] Finally, we will see in section 11.2 (page 91) that amplitude arrows must often be assigned to composite processes, such as the motion of two particles, where it is impossible to associate an amplitude with a single particle.

We have uncovered the second — and last — central concept of quantum mechanics: *The probabilities of various outcomes arise through the interference of amplitudes.* This is a good place to summarize our entire discussion.

A summary of all quantum mechanics

The question of quantum mechanics:

What is the probability of going from one state to another?

The framework for answering that question:

(1) Enumerate all ways of going between the two states.
(2) Assign an amplitude (an arrow) to each way.
(3) Add up all the arrows (place arrows tail to head, the sum stretches from the first tail to the last head).
(4) The probability is the square of the magnitude of this sum arrow.

This list is a framework rather than an actual recipe for answering the question because it doesn't say how to perform the assignment of

[†] Indeed, it is possible to find schemes for calculating the outcome probabilities that do not make use of amplitude arrows at all. One such scheme — which is somewhat like the "water wave" scheme for calculating the interference effects of light — was invented by David Bohm.

[‡] This is not so unusual as you might at first think. For example, the relationship between lengths and positive numbers is not unique. The same length is represented by both 2 (feet) and 24 (inches).

amplitudes to ways required by point (2). Physics majors spend many years learning the rules for assigning amplitudes. (As well as learning how to guess which rules might apply in situations that have not yet been encountered!) For this book, I will just tell you the appropriate rules as they are needed. (If you ask your physicist friends about rules for assigning amplitudes, they won't know what you're talking about. That's because they use the technical phrase "the Hamiltonian (or the Lagrangian) for the system" instead of the phrase "the rules for assigning amplitude arrows".)

Another problem with implementing this framework is less obvious. What, precisely, is meant by a "state"? This is another question that can require considerable thought and experimentation to answer, and for which the answer is sometimes surprising. For the "unwatched" atoms considered so far in this chapter, the state is specified as, for example, "an atom leaving the − exit of the vertical analyzer". But for "watched" atoms, the state specification must give information about both the atom *and* the photon that interacted with it. This is how the results of quantal interference experiments 9.4 through 9.6 on pages 67–68 can be worked into this framework.

For example, part A of figure 10.1 (next page) shows an atom with $m_z = +m_B$ entering an interference apparatus while a photon approaches branch a to observe the atom. (In the figure, the atom is represented by a dot and the photon by a square.) If the atom is observed to pass through branch a (photon is deflected, as in situations B and C) then the intermediate atom has $m_x = +m_B$ and the atom could leave through either the + or the − exit of the vertical analyzer. If the atom is not observed (the photon misses, as in situation D) then the intermediate atom has $m_z = +m_B$ and the atom must leave through the + exit of the vertical analyzer. Thus there is some amplitude to go from state A to state B, and some amplitude to go from state A to state C, but no amplitude to go from state A to state D. But states B and D are exactly the same as far as the atom is concerned, they differ only in the photon. Thus to specify a "state" in this circumstance you must give the position of both the atom and the photon.

Finally, the framework is imprecise about the meaning of "way". Suppose an atom moves from point A to point B. This could be done through a direct, straight line route, or it could be done via a detour to London. Both of these paths are "ways" to perform the move and both must be considered. But there are other, less obvious, ways. For example, the atom could leave point A, move toward B, emit a photon, move toward B a little more, reabsorb that same photon, then continue its journey on to point B. Or it could leave point A intact, break into three pieces and then reassemble before getting to point B. Do such bizarre mechanisms constitute "ways to go from the initial to the final state"? Yes they do. Most of the time, however, such truly bizarre ways can be ignored for practical purposes

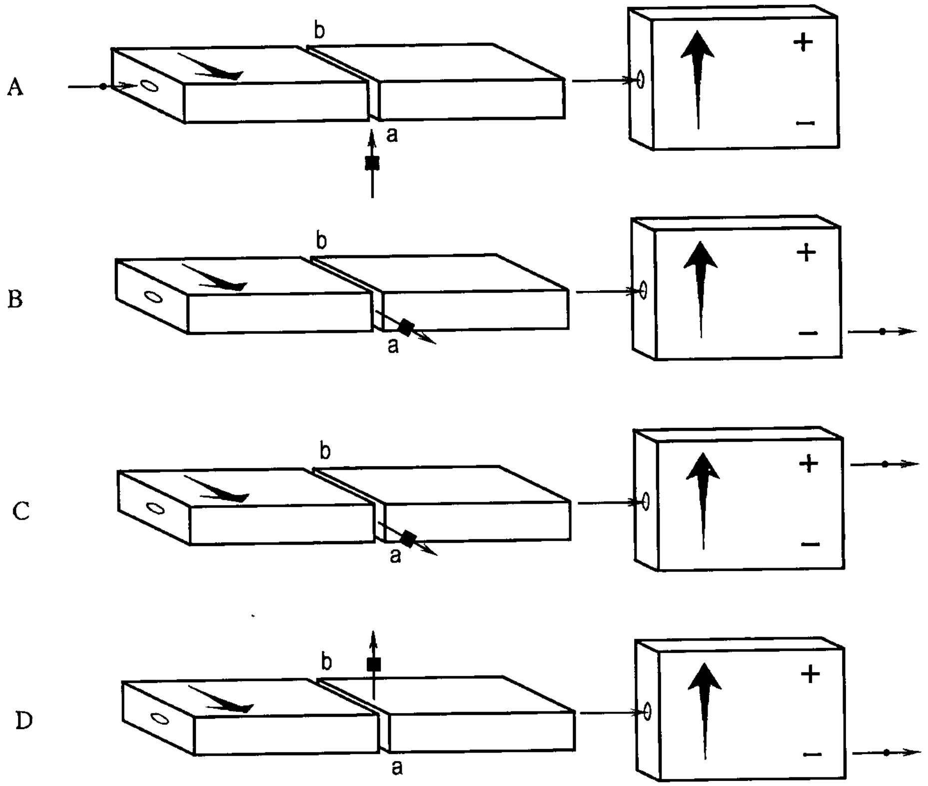


Fig. 10.1. Various states for an atom being observed as it passes through an interferometer. To specify a state, you must give the position of the photon (represented by a square) as well as the position of the atom (represented by a dot).

because (1) the arrows associated with such ways are quite small indeed and (2) there are a host of other ways that are similar to, say, the three-pieces way (for example, the atom breaks into four pieces) and the various arrows from this host of similar ways point in all different directions, so when they are all added together they tend to cancel each other out.

10.2 Evidence for the amplitude framework

In the Einstein–Podolsky–Rosen experiment we found a single definitive§ experiment which proved that classical mechanics (or any other local

§ That is, definitive except for the considerations mentioned on page 49. It is a characteristic of science that all experiments involve error and thus that no experiment — and no scientific statement — is *absolutely* definitive.

deterministic scheme) must be incorrect. It would be nice to present now a definitive experiment which proves that the amplitude framework is correct. *This cannot be done.* One experiment can prove a general idea wrong, but no number of experiments can prove that idea right. This is the nature of a general idea: it is supposed to work in all cases, so if it fails in a single test it must be wrong, but if it passes a million tests it might still fail the million and first test. (To prove that rhinoceroses exist, you only need to find one rhinoceros. To prove that unicorns do not exist, you need to scour the earth and find none.) Because general ideas cannot be proven correct, I will instead present an overview of the many and various situations to which the amplitude framework has been applied, and for which it has never yet been found wanting.

object	approximate size
person	10^{0} meter
fly	10^{-2} meter
hair width	10^{-4} meter
bacterium	10^{-6} meter
DNA width	10^{-8} meter
atom	10^{-10} meter
	10^{-12} meter
nucleus	10^{-14} meter
	10^{-16} meter
	10^{-18} meter
quark	10^{-20} meter
	10^{-22} meter
	10^{-24} meter
	10^{-26} meter
	10^{-28} meter
	10^{-30} meter
	10^{-32} meter
Planck length	10^{-34} meter
	10^{-36} meter

I approach this overview through the above list of objects of various sizes. A person is about two meters tall, so a person is listed on the length scale of 1 meter = 10^{0} meter. (Of course, not all people are the same size, and even if they were, two meters is not the same as one meter. But this list is just a rough guide. This table goes down to objects that are much smaller than atoms, and the basic point — that people are a whole lot bigger than atoms — is made whether people are listed as about one meter tall or about two meters tall.) The list goes to smaller and smaller lengths until it reaches microscopic objects that were not discovered until the end of the nineteenth century. There is a wide range of lengths here — a person is a million times bigger than a bacterium — but classical mechanics is able to explain phenomena at all these length scales.

But here the domain of classical mechanics ends. The structure of atoms was under intense investigation in the 1910s and 1920s, and everyone's first thought was of course to apply classical mechanics to these new length scales. Everyone did, and the results were catastrophic — classical mechanics made a number of patently incorrect predictions about atomic phenomena. Physicists first attempted to work within the framework of classical mechanics by invoking new force laws within the old framework to explain the new observations. These attempts failed. Then they tried to make the smallest possible modifications of the classical framework. Eventually these attempts failed also, and physicists were forced to develop the entire new framework of quantum mechanics to explain these facts. It took a long time growing, but once it arrived the amplitude framework, coupled with rules for assigning amplitudes, was able to explain atomic phenomena with extraordinary accuracy.

The story does not stop here, however. In the 1930s physicists probed the even smaller world of the atomic nucleus. Many strange and wonderful phenomena were uncovered. There was talk that quantum mechanics would not be able to explain these new observations, and that it would have to yield to yet another framework. But no: after sufficient thought and experimentation it was found that the amplitude framework was adequate for explaining nuclear phenomena, although new rules for assigning amplitudes had to be developed.

In the 1950s and 1960s the subnuclear world was investigated in detail. New elementary particles were discovered, new and strange interactions were found, and there was talk that a new version of mechanics would be necessary to explain all the observations. But after a while it was found that the quantal framework was perfectly adequate for the subnuclear world, once the proper rules for assigning amplitudes were uncovered. Now the nucleus is known to be made of neutrons and protons, which in turn are made up of quarks. Studies of quarks have led to measuring the shortest length ever experimentally investigated, about 10^{-19} meter. This length is as small, relative to an atom, as an atom is small, relative to a person. All the way down this staircase, the framework of quantum mechanics has proved to be adequate.

But while experimentalists — for now — cannot look smaller than 10^{-19} meter, there is nothing to stop theorists from speculating about even shorter length scales. Right now a lot of theoretical investigation centers on lengths around 10^{-35} meter, the so-called Planck length, where quantum effects become important for the gravitational force. The Planck length is even smaller, relative to a nucleus, than a nucleus is, relative to a person. In the 1980s theorists started to do calculations concerning phenomena at this length scale, and all sorts of impossible things started to come out. There was talk that a new framework of mechanics would be needed to replace the quantal framework, but eventually new rules for assigning amplitudes were found that enable calculations to be performed consistently. These new rules go under the name of "superstring theory", and they are very strange indeed: They predict a universe of nine spatial dimensions, six of which have curled up into little tubes so tiny that we don't notice them. (In fact, the little tubes are *so* tiny that atoms don't notice them either.) They describe a world where every particle has a complementary "sparticle", and where elementary particles themselves are more like threads or handkerchiefs than like dots. Strange as this theory is, however, its newness falls entirely within the domain of rules for assigning amplitudes — it employs exactly the same quantal framework that was uncovered in the 1920s.

In short, the framework of quantum mechanics has proven to be remarkably resilient, capable of explaining phenomena all the way from 10^{-10} meter to 10^{-35} meter. (In fact it also explains phenomena at lengths above the atomic scale, because these phenomena are governed by classical mechanics and, as we mentioned briefly in chapter 1 and will see in more detail in chapter 14, classical mechanics is nothing but an approximation to quantum mechanics that is accurate only at large length scales.) It has often happened that new amplitude rules were needed to explain the new phenomena discovered when a new length scale was investigated, but so far such new rules have always slipped seamlessly into the amplitude framework.

What of the future? We can expect that physicists will keep on investigating new phenomena. We can expect that new rules for assigning amplitudes will be uncovered. Will these new rules always fit into the by-now-familiar framework? It is of course impossible to know what will happen when these investigations are carried out, but my own guess is that the quantal framework is *not* the final word. My guess is that at some point someone will investigate a phenomenon — perhaps a newly discovered one, perhaps an old one that hadn't received the attention it deserved — and find that it cannot be fit into the quantal framework, no matter how hard scientists attempt to force it in. When that happens, a new framework will have to be developed. If you don't like quantum mechanics, this might make you happy, but watch out. It is my guess that this new framework will seem, to our classical sensibilities, even further away from common sense, even less intuitive, even stranger, than quantum mechanics.

10.3 References

One of the best descriptions of the amplitude framework is

R.P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton, New Jersey, 1985) pages 36–76.

Three wonderful illustrated guides to length scales are the short film

Powers of Ten: A film dealing with the relative size of things in the universe and the effect of adding another zero directed by Charles and Ray Eames (Pyramid Films, Santa Monica, California, 1978),

the associated book

Philip and Phylis Morrison, *Powers of Ten: A book about the relative size of things in the universe and the effect of adding another zero* (Scientific American Library, San Francisco, 1982),

and the CD-ROM (which includes the film)

Powers of Ten Interactive by Demetrios Eames and the Eames Office (Pyramid Media, Santa Monica, California, 1999).

The Oscar-nominated large-screen motion picture

Cosmic Voyage directed by Bayley Silleck (Imax Corporation, 1996)

treats the same material in a very different way. If you ever have a chance to see this film, then do so, you won't regret it. A scholarly and historical (but still very readable) survey of length scales is

Abraham Pais, *Inward Bound: Of Matter and Forces in the Physical World* (Clarendon Press, Oxford, UK, 1986).

The deepest experimental plunge into smallness — the investigation at the scale of 10^{-19} meter mentioned in this chapter — is described in

F. Abe *et al.*, "Measurement of dijet angular distributions by the collider detector at Fermilab", *Physical Review Letters*, **77** (1996) 5336–5341.

Superstring theory is treated at the popular level by

Michio Kaku, *Hyperspace* (Oxford University Press, New York, 1994),

Timothy Ferris, *The Whole Shebang: A State-of-the-Universe(s) Report* (Simon and Schuster, New York, 1997),

Michael J. Duff, "The theory formerly known as strings", *Scientific American*, **278** (February 1998) 64–69.

10.4 Problems

10.1 *Barriers to understanding.* (Compare problem 4.11.) Distinguish between "a description of quantum mechanics", "an understanding of quantum mechanics", and "an explanation for quantum mechanics".

10.2 *Logical contradiction vs. unfamiliar visualization.* For the magnetic needle of a silver atom, we found that

> If *the atom's magnetic needle were just like a classical arrow,* then *the conundrum of projections* would be much worse than a puzzle, it would be a logical contradiction. We are able to regain logical consistency only by abandoning the mental picture of *a magnetic needle as a pointy stick.*

Change the three phrases in italics to produce a parallel statement concerning the position of an atom.

10.3 *States of observed atoms.* Demonstrate that in figure 10.1 we cannot give an amplitude for the atom to move from one place to another, but we must instead give an amplitude for the atom and the photon to move from their two initial positions to their two final positions.