8
Optical Interference

8.1 Overview

We have uncovered the first central principle of quantum mechanics, which is that *the outcome of an experiment cannot, in general, be predicted exactly; only the probabilities of the various outcomes can be found.* In particular, for the magnetic arrow of a silver atom, we know:

> If m_z has a definite value, then m_x doesn't have a value. If you measure m_x, then of course you find some value, but no one (not even the atom itself!) can say with certainty what that value will be — only the probabilities of measuring the various values can be calculated.

How do you like it? Do you feel liberated from the shackles of classical determinism? Or do you feel like Matthew Arnold, who wrote in *Dover Beach* that

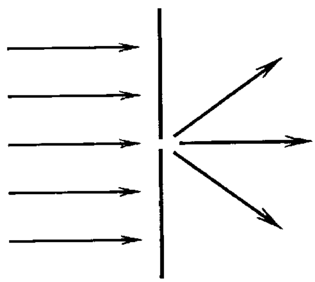
> ... the world, which seems
> To lie before us like a land of dreams,
> So various, so beautiful, so new,
> Hath really neither joy, nor love, nor light,
> Nor certitude, nor peace, nor help from pain;
> And we are here as on a darkling plain
> Swept with confused alarms of struggle and flight,
> Where ignorant armies clash by night.

Regardless of your personal reaction, it is our job as scientists to *describe* nature, not to dictate to it!

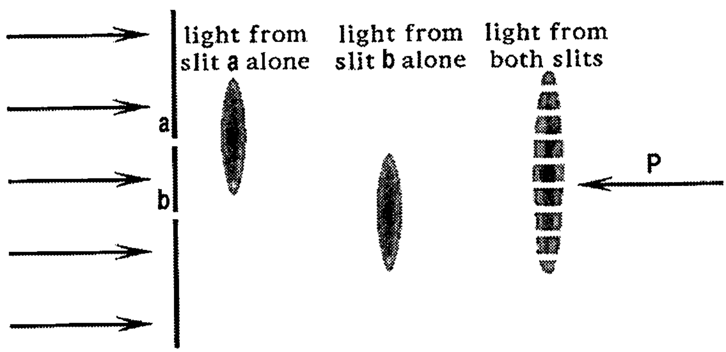
In particular, we know that the model of a magnetic needle as an arrow, so carefully developed in chapter 2 and so correct within the domain of classical mechanics, must be wrong. In classical mechanics, magnetic needles behave like pointy sticks that precess in uniform magnetic fields and that both precess and move in non-uniform magnetic fields. We know that they don't behave this way in quantum mechanics, but we don't yet know how they *do* behave. We begin our search for their true behavior by examining what will turn out to be an analogous phenomenon, namely interference in light.

8.2 The interference of light

Light does not always travel in straight lines. You can demonstrate this for yourself with no more equipment than your hand and a street lamp. Go out on a dark night and look at the street lamp through a V formed by two of your fingers. Bring your fingers closer together to close the gap of the V. Just before your fingers touch and totally block your view of the street lamp, you will see the image of the street lamp become wider and wider as the gap between your fingers becomes narrower and narrower. (Alternatively, squint at the street lamp — as your eye lids grow very close together, the image of the street lamp grows very broad.) This is because light "spreads out" when it passes through a very narrow slit:



Even more remarkable is what happens when light passes through *two* adjacent narrow slits, a phenomenon called "two-slit interference":



If only slit a were open, the light would spread out, as we have just seen, to make a wide bright band centered behind slit a; similarly for slit b. But if both slits were open, then the light would break up into a number of narrow, very bright bands separated by complete darkness. Notice particularly the situation at point P: This point is bright if either slit a or slit b is open, but dark if both slits are open! The term "interference" is quite appropriate for the phenomenon at this point: the light coming from slit b does not "cooperate" with the light from slit a to make brightness at point P, instead it "interferes" with the light provided by a to produce darkness.

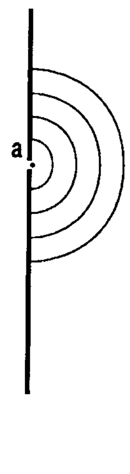
You can demonstrate interference at home also, although the demonstration applies to light passing through many slits rather than through just two slits. (The results for the two different cases are actually quite similar.) A feather contains many parallel narrow slits. If you view a street lamp through a feather, you will see several images of the street lamp located side by side, and separated by darkness at points like point P.

Any explanation/description/recipe for this phenomenon must allow two sources of light to add up to darkness. I will describe two possibilities: the imaginary undulation and the imaginary stopwatch hand.

8.3 The undulation picture

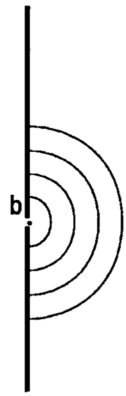
In this picture each slit acts as a source of imaginary undulations — like water waves except that there's no water. The undulations are close together: 15 800 wave crests per centimeter for red light, somewhat more for other colors. The total "water surface motion" is the sum of the undulations from each source. The sensation of light brightness at a point is due not to the height of the "water" there, but due to the difference in height from crest to trough there. (Quantitatively, in fact, the brightness is proportional to the square of that difference.)

The figure below is a schematic diagram of the imaginary undulations

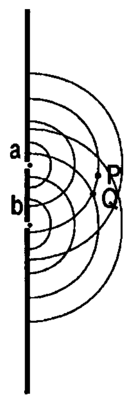


produced by a single slit at a given instant. The thin lines mark the wave crests, the troughs (not shown in the diagram) fall half-way between the crest lines. As time goes on the wave crests travel to the right, and new crests emerge from the slit source to replace them. (At the particular instant shown in this snapshot, a new crest is just about to emerge from the slit.)

The figure below similarly shows the imaginary undulations produced by a different single slit, located somewhat lower.



What happens when both slits are present? The figure below is similar to the two above in that it shows the situation at a single instant, but it differs in that the circles radiating from the two slits do not mark wave crests. Instead the circles radiating from slit a mark where the wave crests would be if only slit a were present, and those radiating from slit b mark where they would be if only slit b were present. The actual status of the water surface, i.e. the total "water surface motion", must be found by summing the undulation from slit a and the undulation from slit b.

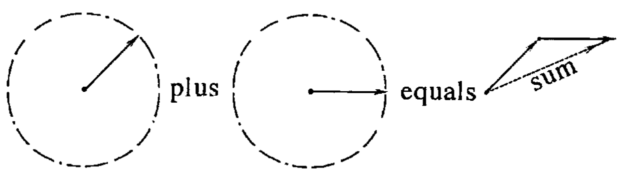


Point Q is located exactly half-way between slit a and slit b, so when a crest from a arrives there a crest from b arrives also, and the "water level" is very high. Similarly the troughs from a reach point Q at the same time that the troughs from b do, so then the "water level" is very low. Thus the "water surface" rises and falls dramatically at point Q, corresponding to intense brightness there.

The situation at point P is very different. It is somewhat closer to slit a than it is to slit b, so when a crest from slit a arrives, the corresponding crest from slit b is still in transit, instead the contribution from slit b is the preceding trough! At point P the crests from a arrive on top of the troughs from b, and the troughs from a arrive on top of the crests from b. Indeed, the contributions from the two slits exactly cancel out at all times, so the "water surface" does not move at all, corresponding to complete darkness. Now you can see how, in this picture, two sources of light can interfere to produce darkness.

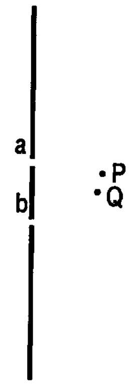
8.4 The stopwatch hand picture

In this picture each slit sends out streams of "photons" ("particles of light"). When the slit releases a photon, an imaginary stopwatch hand starts moving. For red light, the hand rotates 15 800 times every time the photon moves one centimeter. To find the brightness at any point, add the two stopwatch hands (one from each slit) by laying them tail to head. The "sum" stretches from the tail of the first stopwatch hand to the head of the second stopwatch hand. The brightness at that point equals the square of the magnitude (i.e. the "length") of the sum.

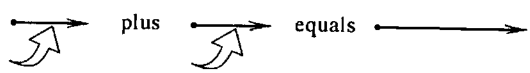


It is traditional to start each stopwatch hand pointing to the right, i.e. to 3 o'clock, and to rotate it counterclockwise, but this is only convention. Any other convention, as long as it is applied consistently, will find the same resulting brightness pattern. A stopwatch hand is also called a "rotating arrow", a "phasor", or, by the cognoscenti, a "complex number".

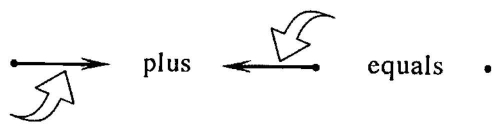
For example, in the two-slit situation described above, the stopwatch hand associated with photons that travel from a to Q starts pointing at 3 o'clock, rotates three times, and ends up pointing to 3 o'clock again. (This is because the distance from a to Q is exactly 3/(15 800) centimeters.) Similarly for the stopwatch hand associated with photons that travel from



b to Q (which is exactly the same distance). The sum arrow is a long one, corresponding to intense brightness.



Turning our attention now to point P, we find that the arrow associated with photons traveling from a to P rotates three complete times, stopping at 3 o'clock. (The distance from a to P is again 3/(15 800) centimeters.) Meanwhile the arrow associated with photons traveling from b to P rotates three and a half times, stopping at 9 o'clock. (Because the distance from b to P is 3.5/(15 800) centimeters.) These two arrows add to zero,



corresponding to complete darkness. Again you see how this picture, like the wave picture, permits two sources of light to interfere and produce darkness.

8.5 Philosophical remark

There are no stopwatch hands, just as there is no water. Both of these pictures are nothing but analogies — mathematical schemes that permit us to calculate the brightness of the light striking various points. Yet both pictures give complete and accurate descriptions of the behavior of light. One scheme cannot be preferred over the other on scientific grounds, because both give exactly the same results for the brightness. Neither scheme gives an underlying mechanism* that tells us "what's really going on". Neither, I suppose, is what God was thinking when he/she/it created the universe. If you want the answers to such questions, you must consult a priest, not a scientist.

8.6 Problems

8.1 *Adding arrows.* Three stopwatch hour hands each have a length of five inches. One stopwatch hand points to noon ("due north"), the second to 3 o'clock ("due east"), and the third to 1:30 ("northeast"). How long is the sum of the three arrows, and in which direction does it point? Hint: This is the $1{:}1{:}\sqrt{2}$ right triangle again.

8.2 *Philosophical remark.* Here are three different ways to add seven and sixteen: (1) Use arabic numerals 7 and 16. (2) Use roman numerals VII and XVI. (3) Put seven marbles in a box, put in sixteen more, then count all the marbles in the box. Which process is "really going on" in addition?

* For an insightful discussion about mathematical algorithms *vs.* clockwork mechanisms, see chapter 2, "The relation of mathematics to physics" of R.P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, Massachusetts, 1965).