# 7

# Variations on a Theme by Einstein

The previous chapter covered the most important aspects of the Einstein–Podolsky–Rosen conundrum. But some interesting new features have come up since Aspect performed his experiments, and I thought you might enjoy them, so I'll mention two of them here. You may skip this chapter without interrupting the flow of the book's argument.

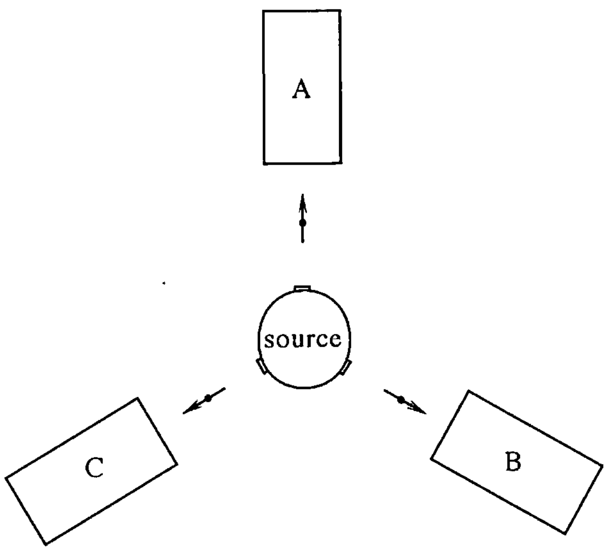
The results of the Aspect experiment were welcomed by most scientists as a final confirmation of the principles of quantum mechanics, principles that had already been verified magnificently in numerous experiments that were not as clean nor as easy to understand as the test of Bell's theorem. But scientists also looked for possible flaws in the confirmation, and they found one. We have discussed an ideal experiment, in which the source produces a pair of atoms and each tilting analyzer detects one of them. But in Aspect's real experiment, it often happened that after the source launched its atoms only one of the two atoms was detected, and sometimes neither of them were. This is not surprising: perhaps one of the atoms collided with a stray nitrogen molecule and was deflected away from its detector, or perhaps the detector electronics were pausing to reset after detecting one atom when a second atom rushed in. For these reasons, in analyzing his experiment Aspect ignored cases where only one atom was detected. But another possibility is that each atom is generated with an instruction set which could include the instruction "don't detect me". If this possibility is admitted, then one can invent local deterministic schemes that are consistent with Aspect's experimental results.

Personally, I regard this objection as far-fetched. But either of the two proposed experiments described here would overrule this objection definitively, because both of them produce situations in which quantum mechanics predicts that something might happen, whereas local determinism predicts that the same thing will *never* happen. Neither experiment has been executed in its entirety, but work is in progress on both and the preliminary results announced to date support quantum mechanics and oppose local determinism.

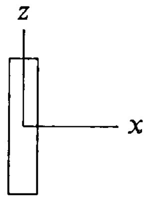
## 7.1 The Greenberger–Horne–Zeilinger variation on the Einstein–Podolsky–Rosen experiment

This experiment involves a source that ejects three atoms in an initial state that is hard to produce and even harder to describe. It is impossible for me to justify the prediction of quantum mechanics in a book at this level. For these two reasons I considered ignoring this experiment altogether in writing this book. But there is a payoff so rich that I had to include it: Whereas the test of Bell's theorem gives a circumstance in which the quantal probability for something happening is 50% while the local deterministic probability is more than 55%, the Greenberger–Horne–Zeilinger (or GHZ) variation gives a circumstance in which the quantal probability is 1 and the local deterministic probability is 0.

A top view of the Greenberger–Horne–Zeilinger experiment is sketched below. The source ejects three atoms in a special state, and each atom flies




off to its own detector. Like the detectors in the test of Bell's theorem, each box contains a Stern–Gerlach analyzer that can be tilted and set to various orientations. But unlike the tilting analyzers used before, these analyzers can be set to only two orientations: the $z$ direction (vertical) or the $x$ direction (horizontal).




Back panel of each Greenberger–Horne–Zeilinger detector, showing the two orientations for its internal Stern–Gerlach analyzer. This analyzer is set to orientation $z$.

The orientations of the three analyzers are reported through a code like $xxz$, which means that detectors A and B are set to $x$ while detector C is set to $z$. As with the test of Bell's theorem (experiment 6.2, page 42), the detector orientations can be set after the atoms have been launched, while they are still in flight toward the detectors.

The predictions of quantum mechanics are:

| | detector settings | what happens |
|---|---|---|
| (1) | $zxx$ | odd number (1 or 3) go to + |
| (2) | $xxz$ | odd number (1 or 3) go to + |
| (3) | $xzx$ | odd number (1 or 3) go to + |
| (4) | $zzz$ | even number (0 or 2) go to + |
| (5) | other | not used in this argument |

Thus whenever two analyzers are set to $x$ and one to $z$, either all three atoms leave through the + exits of their respective analyzers, or else one leaves through the + exit and the other two leave through − exits.

### *The argument for instruction sets*

I will give an argument based on line (1) of the prediction that makes it seem reasonable that each atom is launched from the source with an instruction set, so that it will know whether to go to + or to − when it reaches its detector, regardless of what the settings of the detectors are. If you find this assertion reasonable already, you may skip the argument. Remember, however, that quantum mechanics maintains that this natural surmise is *not* correct, because an atom with a definite value of $m_x$ does not have a definite value of $m_z$.

Suppose that I wished to measure the value of $m_x$ for the atom going to detector C. One way to do it would be by setting A to $z$, B to $x$, and C to $x$, corresponding to line (1) of the prediction. Then I ask what would happen if I used only the detectors at A and at B, and forgot about the detector at C. (This despite the fact that it is the atom going to C that I'm interested in.) If detector A (set to $z$) measured +, and detector B (set to $x$) measured +, then detector C (set to $x$) would have to measure + as well, because according to line (1) of the prediction there must be either one or three atoms going to +. So if the atoms going to A and B come out through the + exit, then I don't need to actually measure $m_x$ of the atom going to C — I know what's going to happen at C merely by observing what had happened at A and B.

In fact, the same is true regardless of how the atoms come out at A and B, as long as the detectors are set to $zxx$:

| | outcomes at A | outcomes at B | | outcome at C |
|---|---|---|---|---|
| | + | + | | + |
| given | + | − | then | − |
| | − | + | | − |
| | − | − | | + |

In short, if the settings are $zxx$, then by reading the outcomes at A and B, I can determine the outcome at C. I don't need to actually put an analyzer at C. The same is true for other directions: reading the outcomes at B and C enables me to determine the outcome at A, and reading the outcomes at A and C enables me to determine the outcome at B. And a glance at the quantal prediction on page 51 will convince you that parallel statements hold if the settings are $xxz$ or $xzx$. In short, lines (1), (2), and (3) of the prediction enable you to determine either $m_z$ or $m_x$ of any atom merely by measuring appropriate quantities for the other two atoms, without actually touching the atom in question.

Because the detectors don't communicate with each other, the natural interpretation of this fact is that when an atom is launched from the source, it must already "know" how it will behave at the detector, regardless of the setting of that detector. Such an "instruction set" might be encoded into the direction of the atom's magnetic arrow, but it could conceivably be encoded in some strange or complicated way. In what follows I make no assumption about how the instruction set is encoded, only that it exists.

### *The prediction of local determinism*

I will write down the instruction set of all three atoms using a symbol like

$$\begin{array}{c l} \text{atom heading toward} & \\ \begin{array}{ccc} \mathrm{A} & \mathrm{B} & \mathrm{C} \end{array} & \\ \begin{pmatrix} + & - & - \\ - & - & + \end{pmatrix} & \begin{array}{l} \leftarrow \text{if set to } z \\ \leftarrow \text{if set to } x \end{array} \end{array}$$

This notation means that the atom heading toward detector A will leave through the + exit if that detector is set to $z$, through the − exit if it is set to $x$. The atom heading toward detector B will leave the $-$ exit regardless of setting. The atom heading toward detector C will leave the $-$ exit if that detector is set to $z$, the $+$ exit if it is set to $x$.

Now I ask: What instruction sets are consistent with the quantal prediction? We will examine the first four lines of the table on page 51 in turn.

Line (1) of the table pertains to detector settings $zxx$, so it has nothing to say about what will happen if A is set to $x$, if B is set to $z$, or if C is set to $z$. In the following table the instructions for such settings are set to "?". Notice from the table that with these settings either one or three atoms leave through the $+$ exit, and therefore the only instruction sets compatible with line (1) are the following:

| instruction sets consistent with line (1) | |
|---|---|
| $\begin{pmatrix} + & ? & ? \\ ? & - & - \end{pmatrix}$ | $\begin{pmatrix} - & ? & ? \\ ? & + & - \end{pmatrix}$ |
| $\begin{pmatrix} - & ? & ? \\ ? & - & + \end{pmatrix}$ | $\begin{pmatrix} + & ? & ? \\ ? & + & + \end{pmatrix}$ |

Which of these instruction sets is consistent with line (2) of the quantal prediction as well? We begin by considering only instruction sets of the type shown in the upper left above. Line (2) involves the setting $xxz$, so this reasoning will enable us to fill in the $x$ (bottom) slot of column A and the $z$ (top) slot of column C. We already know, from the entry above, that the atom heading for detector B will come out through the $-$ exit. Since a total of either one or three atoms must come out through the $+$ exit in this circumstance, then of the atoms heading for A and C, one must come out through $+$ and the other through $-$. Thus the instruction set must be either

$$\begin{pmatrix} + & ? & + \\ - & - & - \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} + & ? & - \\ + & - & - \end{pmatrix}.$$

The same game can be played with the other three types of instruction sets consistent with line (1), resulting in:

| instruction sets consistent with lines (1) and (2) | | | |
|---|---|---|---|
| $\begin{pmatrix} + & ? & + \\ - & - & - \end{pmatrix}$ | $\begin{pmatrix} + & ? & - \\ + & - & - \end{pmatrix}$ | $\begin{pmatrix} - & ? & - \\ - & + & - \end{pmatrix}$ | $\begin{pmatrix} - & ? & + \\ + & + & - \end{pmatrix}$ |
| $\begin{pmatrix} - & ? & - \\ + & - & + \end{pmatrix}$ | $\begin{pmatrix} - & ? & + \\ - & - & + \end{pmatrix}$ | $\begin{pmatrix} + & ? & - \\ - & + & + \end{pmatrix}$ | $\begin{pmatrix} + & ? & + \\ + & + & + \end{pmatrix}$ |

From here it is easy to find the instruction sets consistent with line (3) of the quantal prediction as well:

instruction sets consistent with lines (1), (2), and (3)

$$\begin{pmatrix} + & + & + \\ - & - & - \end{pmatrix} \quad \begin{pmatrix} + & - & - \\ + & - & - \end{pmatrix} \quad \begin{pmatrix} - & + & - \\ - & + & - \end{pmatrix} \quad \begin{pmatrix} - & - & + \\ + & + & - \end{pmatrix}$$

$$\begin{pmatrix} - & + & - \\ + & - & + \end{pmatrix} \quad \begin{pmatrix} - & - & + \\ - & - & + \end{pmatrix} \quad \begin{pmatrix} + & - & - \\ - & + & + \end{pmatrix} \quad \begin{pmatrix} + & + & + \\ + & + & + \end{pmatrix}$$

These eight, now completely determined, instruction sets are the only ones consistent with the quantal predictions given in lines (1), (2), and (3).

Which of these eight instruction sets is consistent with line (4) as well? In line (4) the detectors are set to *zzz*, so only the upper row of the instruction sets are relevant. The instruction set shown in the upper left above would result in all three atoms leaving through the + exits of their analyzers. But according to quantum mechanics (see line (4) of the quantal prediction on page 51), in this case an even number of atoms must leave + exits. Three is an odd number, so the instruction set in the upper left above must be ruled out as inconsistent with the predictions of quantum mechanics. The instruction set in the lower right must be ruled out for the same reason. All the remaining instruction sets call for exactly one of the three atoms to leave through + exits. But one is also an odd number! In short:

instruction sets consistent with lines (1), (2), (3), and (4)

NONE!

Once again, the existence of instructions sets — regardless of how subtly the instructions are encoded — is inconsistent with the predictions of quantum mechanics.

## 7.2 Hardy's variation on the Einstein–Podolsky–Rosen experiment

This variation is harder to describe and I will not treat it in detail. It involves a source that ejects two atoms toward two different detectors, each of which can be tilted to two different angles, and an unusual initial state at the source. The experiment looks for a certain combination of events. The local deterministic prediction is that this combination will never happen. The quantal prediction is that it will happen with a

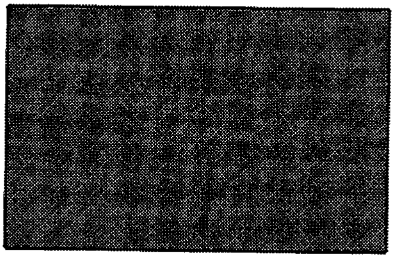


Fig. 7.1. The golden rectangle.

probability of 9.017%. Thus if the combination happens in an experiment even once, then local determinism must be wrong.

One thing that intrigues me about this variation is the mathematical origin of the probability 0.09017.... The number is $g^5$, where the constant $g$ is equal to $(\sqrt{5}-1)/2 = 0.6180\ldots$ and is called "the golden mean". If a line of length 1 is divided into two pieces so that the ratio of the length of the whole to the length of the long piece is equal to the ratio of the length of the long piece to the length of the short piece, then the long piece will have length $g$. The ancient Greeks considered a rectangle of width 1 and height $g$ to be the "ideal" (most beautiful) rectangle. The Parthenon in Athens, for example, has a height of $g$ times its width. Rectangles with these proportions also appear in the work of Leonardo da Vinci, Titian, and Mondrian. In addition the number is connected with the Great Pyramid, the star pentagram (which in one form appears in the American flag and which in another is said to call up the devil), the Fibonacci sequence, recursion relations, and with algorithms for locating the minimum of a one-variable function. But this is the first time I've ever seen it appear in quantum mechanics.

## 7.3 References

For the Greenberger–Horne–Zeilinger variation, see


Daniel M. Greenberger, Michael A. Horne, Abner Shimony, and Anton Zeilinger, "Bell's theorem without inequalities", *American Journal of Physics*, **58** (1990) 1131–1143,

N.D. Mermin, "The (non)world (non)view of quantum mechanics", *New Literary History*, **23** (1992) 855–875,

D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, "Observation of three-photon Greenberger–Horne–Zeilinger entanglement", *Physical Review Letters*, **82** (1999) 1345–1349.

For Hardy's variation, see

Lucien Hardy, "Nonlocality for two particles without inequalities for almost all entangled states", *Physical Review Letters*, **71** (1993) 1665–1668,

David Branning, "Does nature violate local realism?", *American Scientist*, **85** (1997) 160–167.

A technical but insightful exchange concerning Hardy's variation and its implications for locality in quantum mechanics is

Henry P. Stapp, "Nonlocal character of quantum theory", *American Journal of Physics*, **65** (1997) 300–304,

N. David Mermin, "Nonlocal character of quantum theory?", *American Journal of Physics*, **66** (1998) 920–924,

Henry P. Stapp, "Meaning of counterfactual statements in quantum physics", *American Journal of Physics*, **66** (1998) 924–926.