3

The Stern–Gerlach Experiment

3.1 Measuring magnetic projections

What does the previous chapter have to do with quantum mechanics? I have said that the predictions of quantum mechanics are significantly different from those of classical mechanics only when applied to very small objects. How could we make such a tiny compass needle? In fact we don't need to make one, because nature itself supplies one. It is natural to suppose that an atom acts like a tiny magnetic needle because its orbiting electron mimics a current loop.

In 1922, physicists Otto Stern and Walther Gerlach decided to test this supposition by measuring the magnetic arrow associated with a silver atom. It is clear that they could not do this by watching an individual atom precess in a uniform magnetic field! Instead, they injected a moving silver atom into a non-uniform field and noticed how the resulting force pushed the atom around. The "Stern–Gerlach apparatus" sketched on the next page thus measures the projection of an atom's magnetic arrow on the vertical axis.

What results would you expect from this experiment? Think about this for a moment before reading on.

3.2 Classical expectations

I don't know about you, but here is what I would expect: Once the atom enters the non-uniform magnetic field, its magnetic arrow precesses in such a way that its projection on the vertical axis remains constant. While this precession is taking place, there is also a force on the atom, and the magnitude of that force depends upon the value of the projection. If the atom has a large positive projection, it will experience a large upward force and move up sharply. If the atom has a small positive projection,

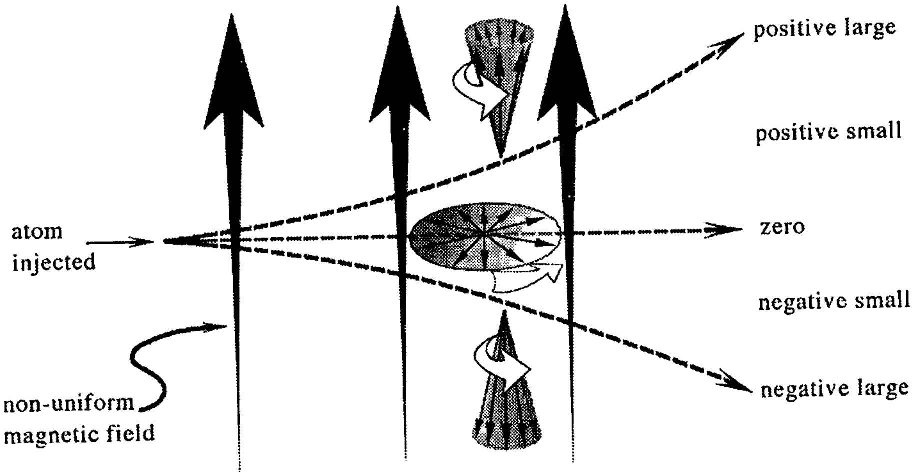


Fig. 3.1. A sketch of the Stern–Gerlach apparatus, with some justification for my classical expectations.

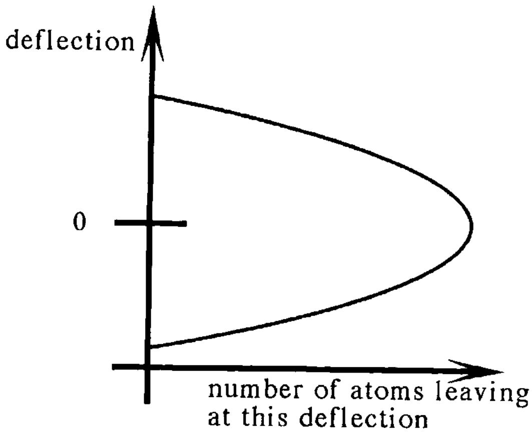
it will experience a small upward force and move up moderately. If the projection happens to be zero, the atom will experience zero force and move straight through. Similarly for atoms with negative projections.

Thus an atom that happened to enter the field with its arrow pointing straight up ("toward the north pole") would experience a large upward deflection. One that happened to enter with its arrow pointing straight down ("toward the south pole") would experience a large downward deflection. And one that happened to enter with a horizontal arrow ("toward the equator") would experience no deflection. Atoms whose arrows had intermediate tilts would experience intermediate deflections.

Now, there is only one way for an arrow to point toward the north pole, and only one way for it to point toward the south pole, but there are lots of ways for it to point toward the equator.* There are, in fact, a few ways to point toward the 10° north latitude line, more ways to point toward the 20° north latitude line, still more for the 30° line, and so on until a maximum is reached at the equator (the 90° line). I expect atoms to enter the apparatus with their magnetic arrows pointing every-which-way: some straight up, some straight down, most somewhere in between. Thus I expect a very small number of atoms to come out with the maximum upward deflection, a larger number to come out with moderate upward deflections, the largest number to come out with zero deflection, and sim-

* For example, directly to the right, directly to the left, directly out of the page, half-way between "to the right" and "out of the page", etc.

ilarly for downward deflections. In short, my classical expectation is that the number of atoms leaving the apparatus with a given deflection should depend upon the deflection in the manner sketched here.

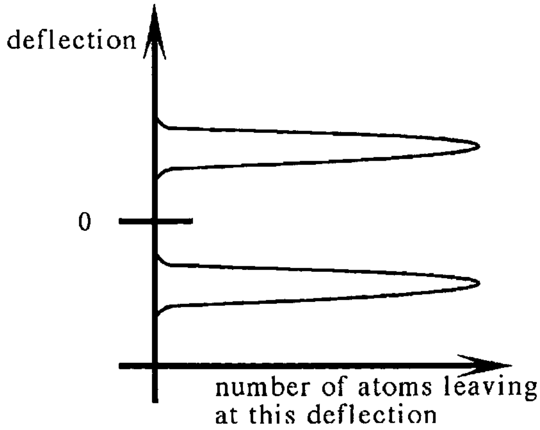


(These expectations are for an ideal experiment. In any real experiment things go wrong — the magnetic field has small imperfections, the source of atoms is not perfectly pure, an atom hits a piece of dust while traveling through the apparatus — so I expect that for a real experiment the curve obtained will be somewhat broadened and stretched away from the results shown above.)

3.3 Actual results

Imagine how surprised Stern and Gerlach must have been when they obtained results that were nothing like the expectations described above. They found that no silver atoms at all went straight through the apparatus. Nor was there a gradual change in number of exiting atoms with deflection. In fact, they found that all of the atoms came out at just two different deflections: one a certain amount up, the other the same amount down. The observed results for silver atoms are summarized in the graph on the next page, where the width of the two humps is due entirely to imperfections in the apparatus.

When atoms other than silver were put through a Stern–Gerlach apparatus, there were sometimes four or five narrow humps, sometimes even more, but never the broad curve of our classical expectations. Furthermore, when the observed deflections were used to compute the values of the magnetic projections, then in all cases, from all different kinds of atoms, the value of that projection turned out to be an integer times a certain quantity called the "Bohr magneton", $m_B = 9.27 \times 10^{-24}$ joule/tesla.



With silver, for example, the two measured values were $+m_B$ and $-m_B$. For nitrogen the four measured values were $+3m_B$, $+m_B$, $-m_B$, and $-3m_B$. For sulfur they were $+4m_B$, $+2m_B$, 0, $-2m_B$, and $-4m_B$. And so on for other atoms.

> *Technical aside:* What if we injected into the apparatus not atoms, but the needles of real live scout compasses? In this case the magnetic projection is huge on an atomic scale — about 0.1 joule/tesla. Presumably the needles will only come out at discrete deflections corresponding to projections of (integer) $\times$ m_B, but instead of giving rise to four or five narrow humps, there will be about 10^{22} of them. There are so many humps, and they are (on the scale of a scout compass) so close together, that the individual humps cannot be distinguished. We find instead a washed out pattern very similar to that of our classical expectations. The principle that when quantum mechanics is applied to big things it must give nearly the same result as classical mechanics is called "the correspondence principle" or "the classical limit of quantum mechanics".

3.4 Actual experiments

I have described the Stern–Gerlach experiment in the simplest possible way so as to focus your attention on the fundamental parts of the experiment rather than on the mundane parts necessary for its operation. But you should realize that this experiment (like any other experiment) is a lot more complicated and a lot more difficult to carry out than the conceptual outline given above. Compare the photo of a real Stern–Gerlach apparatus on the next page to the conceptual outline sketched in figure 3.1!

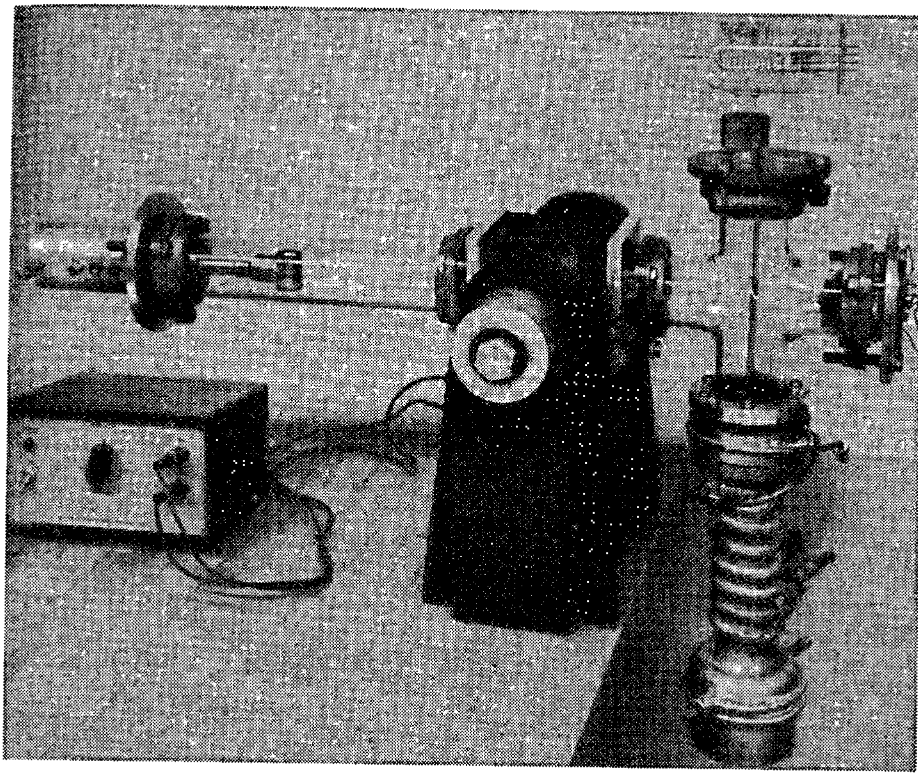


Fig. 3.2. A real Stern–Gerlach apparatus (courtesy of Melvin Daybell).

For example, in our discussion we just said "give us a non-uniform magnetic field" and then we drew it on the page. In the laboratory life is more difficult. Stern and Gerlach had to magnetize two large pieces of iron and carefully shape the pieces so that they would produce the desired magnetic field.

We just said that we needed a source of atoms and a detector of atoms. Stern and Gerlach had to build an electrical oven to eject vaporized silver atoms, and they had to design a suitable detector. (For a detector, they used a glass plate placed to the right of the magnets, and injected enough vaporized atoms that they built up a visible silver deposit on the glass. See figure 3.3.)

We didn't mention at all the possibility that while a silver atom was flying through the magnetic field, it might collide with an oxygen molecule and scatter in some random direction. But Stern and Gerlach had to consider the possibility, so they performed the experiment in a vacuum chamber.

Of course, Stern and Gerlach needed instruments to measure the

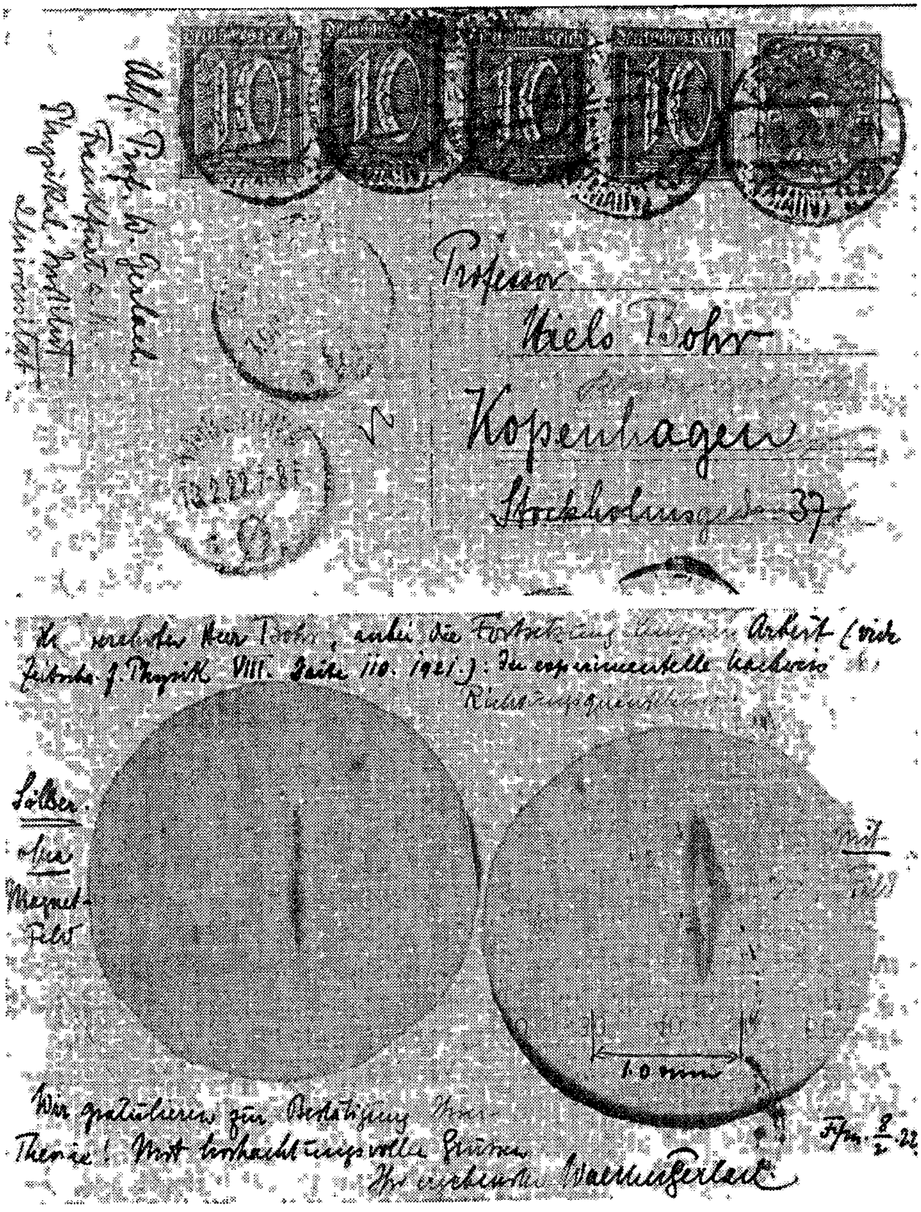


Fig. 3.3. Postcard from Walther Gerlach to Niels Bohr, showing results from one of the earliest, crude Stern–Gerlach experiments. On the left is the beam profile without magnetic field, on the right is the beam profile with a non-uniform magnetic field. Only in the center of the image is the field non-uniformity great enough to pull the two outgoing beams apart. Translation of the message: "My esteemed Herr Bohr, attached is the continuation of our work (vide *Zeitschr. f. Phys.* VIII 110, 1921): the experimental proof of directional quantization. We congratulate you on the confirmation of your theory! With respectful greetings, Your most humble Walther Gerlach." (Courtesy of the Niels Bohr Archive, Copenhagen.)

strength of the magnetic field, the temperature of the oven, and the quality of the vacuum, as well as the number of atoms coming out at a given deflection.

I will mention a number of experiments in this book, and in every case I will present only the simplest conceptual outline. This will keep the concepts clear, but it will ignore a wealth of detail which, while necessary for performing the experiment, serves only to hide the concept. You should be aware that real experiments are always considerably more difficult to perform than thought experiments.

3.5 Visualization

Faced with the unexpected results of the Stern–Gerlach experiment, it is natural to seek a reason for these results: to find a picture that tells us what's going on. There is nothing to be ashamed of in this desire. Human beings are visual animals, and we think best in terms of some picture or visualization that we carry in our minds. Nevertheless I urge you to postpone this quest for a visualization. We will first spend considerable time addressing the question: "We know that silver atoms don't behave exactly like miniature compass needles. Just how do they behave?" Once we know the facts about silver atoms, we will try again (in section 15.2, "What does an electron look like?") to produce an accurate visualization. Seeking a visualization at this point, with our incomplete knowledge, will surely produce a mistaken image. Thomas Huxley described the attitude I am advocating by saying:

> Sit down before fact as a little child, be prepared to give up every preconceived notion, follow humbly wherever and to whatever abysses nature leads, or you shall learn nothing.

3.6 References

A computer program to simulate the Stern–Gerlach experiment is

Daniel V. Schroeder, *Spins*.

You may download this free program (it works on Macintosh computers) through the World Wide Web site mentioned on page xiv.

The history of the Stern–Gerlach experiment is traced in

Immanuel Estermann, "History of molecular beam research: Personal reminiscences of the important evolutionary period 1919–1933", *American Journal of Physics*, **43** (1975) 661–671,

but just as interesting is the story of the intellectual descendents of Stern and Gerlach's work. These descendants include lasers, atomic clocks and the global positioning system, magnetic resonance medical imaging, quantum computers, and a molecule that deactivates the AIDS virus. Some of this richness is described in

> Dudley R. Herschback, "Imaginary gardens with real toads", in *The Flight From Science and Reason*, edited by Paul R. Gross *et al.* (New York Academy of Sciences, New York, 1997) pages 19–24.

3.7 Problems

3.1 *Could friction account for these unexpected results?* Suppose that friction were important for atoms, so that after spending a short time in a magnetic field, all the atomic magnetic arrows would be pointing in the direction of the field. What results would you then expect from the Stern–Gerlach experiment?

3.2 *Real vs. ideal experiments.* How could Stern and Gerlach have known that the width of the peaks they observed was due only to the limits of their instrument and not to some property intrinsic to the atoms? Hint: Examine the figure below.

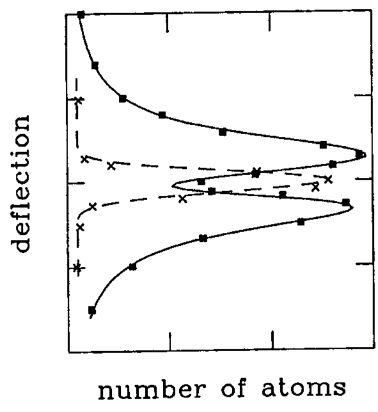


Fig. 3.4. Results from a recent Stern–Gerlach experiment. Solid line and squares show results with non-uniform magnetic field, dashed line and crosses show results with no magnetic field. The vertical scale is magnified; the actual deflections span a range of less than two millimeters.