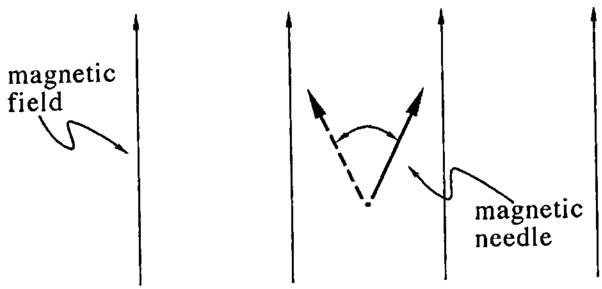
2
Classical Magnetic Needles

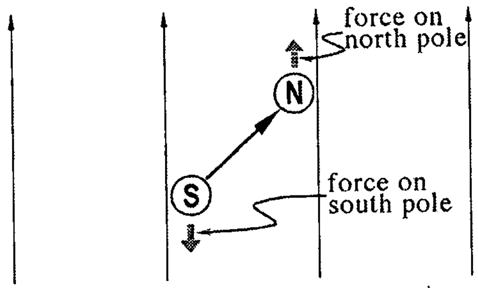
How shall we approach the principles of quantum mechanics? One way is simply to write them down. In fact I have already done that (in the first paragraph of the Preface), but to do so I had to use words and concepts that you don't yet understand. To develop the necessary understanding I will use a particular physical system as a vehicle to propel our exploration of quantum mechanics. Which system? An obvious choice is the motion of a tossed ball. Unfortunately this system, while simple and familiar in classical mechanics, is a complicated one in quantum mechanics. We will eventually get to the quantum mechanics of a tossed ball (in chapter 14, "Quantum mechanics of a bouncing ball", page 103), but as the vehicle for developing quantum mechanics I will instead use a system that is simple in quantum mechanics but that is, unfortunately, less familiar in daily life. That system is the magnetic needle in a magnetic field. This chapter describes the classical motion of a magnetic needle so that we will be able to see how its classical and quantal behaviors differ.

2.1 Magnetic needle in a magnetic field

A magnetic needle — like the one found in any woodsman's compass — has a "north pole" and a "south pole". I will symbolize the magnetic needle by an arrow pointing from its south pole to its north pole. When a magnetic needle is placed in a magnetic field — such as the magnetic field of the earth, or that produced by a horseshoe magnet — then the magnetic field acts to push the north pole in the direction of the field, and to push the south pole in the direction opposite the field. (It is not important for you to understand in detail how this effect works or even what the phrases "north pole" and "magnetic field" mean. Remember that this chapter merely builds a classical scaffolding that will be discarded once the correct quantal structure is built.) These two pushes together twist the needle towards an orientation in which the associated arrow points in the same direction as the field. If the needle starts out pointing parallel to the magnetic field, then it keeps on pointing in that direction. If the needle starts out not pointing parallel to the magnetic field, then it oscillates back and forth about this preferred direction. (If friction is present, then these oscillations will eventually die out and the needle will point precisely parallel to the field. If there is no friction then the oscillations will continue forever. In atomic systems there is no friction.)



If the magnetic field has the same strength at all points in the vicinity of the needle, that is, if the field is uniform, then the upward force acting on the north pole of the needle is exactly cancelled by the downward force acting on the south pole and there is no net force on the needle. So in a uniform field there is an impetus for the needle to oscillate, but no impetus for it to move up or down, or left or right.

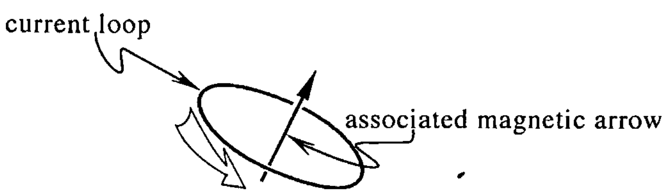


2.2 Magnetic effects on electric current

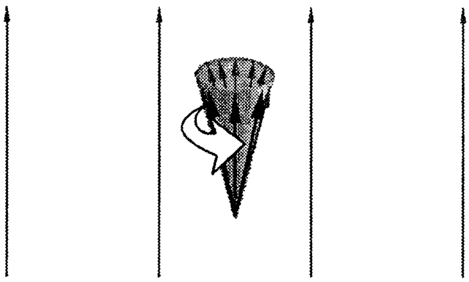
A loop of wire carrying electric current behaves in many ways like a compass needle. The associated magnetic arrow* points perpendicular to the current loop, so if the current loop is placed in a magnetic field, the

* This associated arrow is purely abstract — there's nothing actually located there.

arrow "wants" to point parallel to the field. But the current loop's arrow



isn't *exactly* like a compass needle's arrow, because the current loop arrow *precesses* rather than *oscillates* in a magnetic field. "Precession" means that the tip of the symbolic arrow moves around a circle while its base is fixed. Thus a precessing arrow traces out the figure of a cone. You



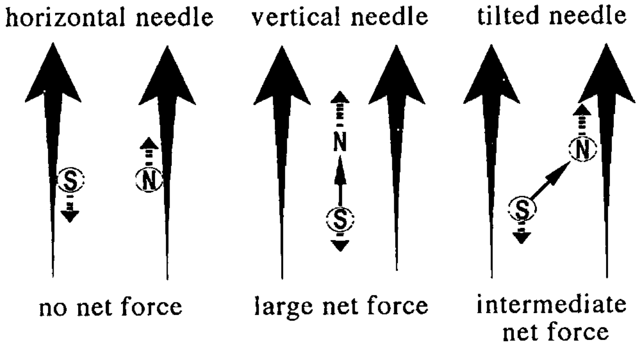
can make your index finger precess by holding it up in the air and then twisting its tip around in a circle while keeping your hand fixed.

I wish I could describe for you an experiment that you could do to prove this fact to yourself. Unfortunately, this cannot be done with the equipment available in the typical home. It is, however, quite easy to do a parallel home experiment with an analogous system. A top rotating in a gravitational field happens to behave very much like a current loop in a magnetic field. (The rotating body of the top is analogous to the moving electric charge — the current — in the loop. The axle of the top is analogous to the magnetic arrow.) I urge you to spin a top, put it on the floor, tip the rotation axis away from the vertical, and then watch the top precess.

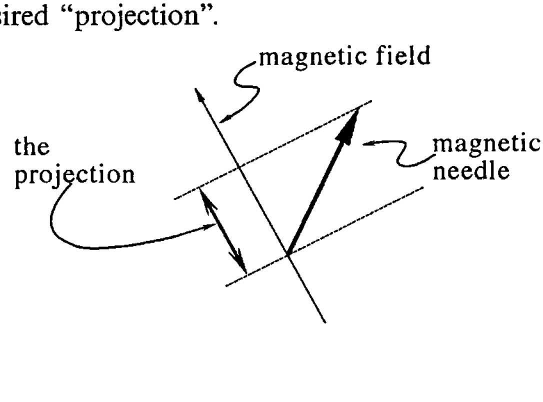
2.3 Magnetic needle in a non-uniform magnetic field

We have seen that a magnetic needle in a uniform magnetic field feels zero net magnetic force, because the upward force on the north pole is cancelled by the downward force on the south pole. But if a magnetic needle is placed in a *non-uniform* magnetic field, then there *can* be a net force on the needle.

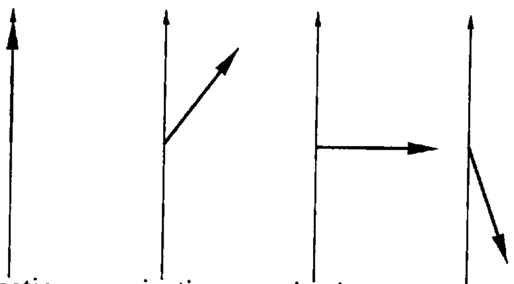
The figure below shows a magnetic field which is stronger at the top of the figure than at the bottom of the figure. For the horizontal needle, both the north and south poles are at the same height and experience the same magnetic field strength, so the two poles experience equal but opposite forces and the net force vanishes. But for the vertical needle, the north pole experiences a stronger magnetic field than does the south pole, so there is a larger upward force on the north pole and a smaller downward force on the south pole. As a result the two forces don't completely cancel — there remains a net upward force. The tilted needle is intermediate between these two situations. It experiences a net upward force, but that force is not as strong as the force on the vertical needle.



You can see that the net force depends upon the angle between the arrow and the field. In fact, the force is proportional to a quantity bearing the awkward name of "the projection of the magnetic arrow onto the direction of the magnetic field". This quantity is defined through a four-stage process: (1) Draw a line to show the direction of the magnetic field (in the illustration below, it tilts to the left). (2) Draw in the magnetic arrow with its base on the field line. (3) Draw a line perpendicular to the field line through the base of the arrow, and another through the tip of the arrow (these are shown dashed). (4) The distance between these two lines is the desired "projection".



Examples of projections:



If an electric current loop is placed in a non-uniform magnetic field, its arrow will precess and at the same time the loop will move. During this precession the projection remains constant,† and hence the force remains constant. For example, suppose the field is stronger at the top than at the bottom (as in the figure on page 8) and suppose a stationary current loop with a small positive projection is placed into the field. Then the current loop will move upward, and as it moves it precesses in such a way that the impetus to move upward stays constant. If the initial projection is negative, then the current loop moves downward.

2.4 Explanation *vs.* description

Have I *explained* the motion of magnetic needles in magnetic fields? Have I *explained* the nature of a magnetic field? Not at all! I have simply *described* these phenomena. Sometimes a description in science can be explained through an appeal to more fundamental principles. For example, I have spoken about the north and south magnetic poles of a compass needle. The poles of a compass needle can in fact be explained in terms of the motion of electrons within the needle's atoms. But in other cases the description is simply the most fundamental thing there is and cannot be "explained" by something else. What is a magnetic field? I have described it, in essence, as "that which makes a compass needle want to oscillate". There are more elaborate and more mathematical descriptions of magnetic field, but none are more fundamental. Science has no explanation for magnetic field, only a description of it.

† Spin your top again and notice that as the top precesses, the tip of the axle remains always the same distance from the floor. The vertical distance from the floor to the tip of the axle is the projection of the axle onto a vertical line. (If you wait long enough that friction slows the rotation of the top, then this projection — the height of the axle tip — will decrease. But if friction can be ignored, then the projection does not change.)

What does "explanation" mean, anyway? Suppose you ask me "Why did it rain yesterday?" I might reply "Because a cold front moved in." Then you could ask "But why did a cold front move in?" I might say "Because the jet stream pushed it." You: "But why did the jet stream push it?" Me: "Because the sun warmed Saskatchewan and so deflected the jet stream."‡ You: "But why does sunlight warm objects?" And at this level I really can't answer your question. I know *that* sunlight carries energy (so do you), and science can describe this energy transport with exquisite accuracy. But science cannot *explain* this energy transport or tell *why* it happens.

This story illustrates that "explanation" means "explanation in terms of something more fundamental". At some point any chain of questioning descends to the most fundamental ideas, and there it must stop. Currently, the most fundamental ideas in physics are called "quantum electrodynamics" and "quantum chromodynamics", two theories which fall squarely within the framework of quantum mechanics that I will describe in this book. Probably there will someday be even more fundamental ideas, so that "why" questions concerning quantum electrodynamics could be answered in terms of these new ideas. However, "why" questions concerning these more fundamental ideas will then be unanswerable! Ultimately, at the bottom of any descending chain of questions, science can only give descriptions (facts) and not explanations (reasons for those facts).

2.5 Problems

> *Above all things we must beware of what I will call "inert ideas" — that is to say, ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combinations.*
>
> — Alfred North Whitehead

Reading books, listening to lectures, watching movies, running computer simulations, performing experiments, participating in discussions … all these are fine tools for learning quantum mechanics. But you will not *really* become familiar with the subject until you get it under your skin by working problems. The problems in this book do not simply test your comprehension of the material you read in the text. They are instead an important component of the learning process, designed to extend and solidify the concepts presented. Solving problems is a more active, and

‡ Anyone who has raised a child is all too familiar with such chains of questions.

hence more effective, learning technique than reading text or listening to lectures.

Some might contend that problems have no place in a book intended for a general audience, because they are "too hard". In fact the opposite is true: it is easier to learn by working problems than by reading words. If "working problems" seems too dry or too regimented for you, then think of it as "solving puzzles" instead.

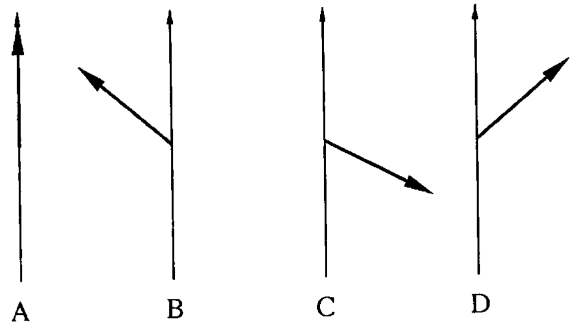
If you write up solutions to these and subsequent problems (such as for a course assignment) be sure to explain your reasoning. Don't just write down the final numerical answer — your teacher already knows what it is! Instead (s)he is interested in seeing how you overcome the roadblocks that get in your way as you progress through the problem. Appendix F (page 149) contains skeleton answers for some of the problems. (There are also three complete sample solutions on pages 28, 71, and 109.) By a "skeleton answer", I mean only the "final numerical answer" mentioned above without any of the reasoning that leads to the answer. I do not present the reasoning because the benefits that accrue from active problem solving come only if you supply that reasoning yourself. The appendix will help you learn quantum mechanics if you work through the problem yourself and then use the skeleton answer to check your reasoning. If you instead look up the answer before attempting the problem, the appendix will actually be an impediment to your learning.

Many of the problems ask for short verbal answers. In all such cases, the answer can be written in four or fewer sentences. If you find yourself writing an extended essay, then you either misunderstood the question or don't know the answer. In neither case will your teacher be impressed by the mere bulk of your response.

> *Technical aside:* This book is intended for a general audience, but it is useful also for students of physics and chemistry who can perform calculations far more sophisticated than anything mentioned here, but who are at a loss to explain what it is that they are calculating. For such readers I have included a few problems that require a more technical background: these problems are clearly marked. Such problems are not harder than regular problems, they just require a background knowledge of physics ideas that general readers are unlikely to possess.

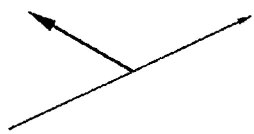
2.1 *Variously tilted needles.* Consider the non-uniform magnetic field of the figure on page 8. Describe the net force acting upon a vertical magnetic needle that points downward, and a tilted magnetic needle that points downward and to the left.

2.2 *Projections on a vertical axis.* The figure below shows four magnetic arrows, labeled A, B, C, and D. All four arrows have the same length. Rank them from highest value of the projection to the lowest. (Do not ignore sign. For example, a large negative projection should rank below a small positive projection.) If two of the projections are equal, then say so.



2.3 *Projections in geography.* The radius of the earth is 3960 miles. The Old Mission Point Lighthouse near Traverse City, Michigan, is located at a latitude of 45°. Imagine an arrow extending from the center of the earth to the Old Mission Lighthouse. How long is the projection of this arrow onto the earth's rotation axis? Hint: Recall a geometrical result about a $1{:}1{:}\sqrt{2}$ right triangle.

2.4 *Projections on a non-vertical axis.* Up to now we have emphasized projections onto a vertical axis. But our definition applies to *any* axis. In the figure below, find the projection of the short, thick arrow on the long, thin axis. Give your answer in inches, with a + or − sign.



2.5 *Different projections on different axes.* Show that for any arrow, you can pick an axis such that the projection of the arrow on that axis is zero. How many such axes are there?

2.6 *The role of mathematics in quantum mechanics.* One of my students wrote "If you can't read music, then you can't write it, but that doesn't mean you can't understand it." To what extent is this analogy appropriate to the use of mathematics in physics?