The Strange World of Quantum Mechanics

This is an exceptionally accessible, accurate, and non-technical introduction to quantum mechanics.

After briefly summarizing the differences between classical and quantum behavior, this engaging account considers the Stern–Gerlach experiment and its implications, treats the concepts of probability, and then discusses the Einstein–Podolsky–Rosen Paradox and Bell's theorem. Quantal interference and the concept of amplitudes are introduced and the link revealed between probabilities and the interference of amplitudes. Quantal amplitude is employed to describe interference effects. Final chapters explore exciting new developments in quantum computation and cryptography, discover the unexpected behavior of a quantal bouncing ball, and tackle the challenge of describing a particle with no position. Thought-provoking problems and suggestions for further reading are included.

Suitable for use as a course text, *The Strange World of Quantum Mechanics* enables students to develop a genuine understanding of the domain of the very small. It will also appeal to general readers seeking intellectual adventure.

DAN STYER is Professor of Physics at Oberlin College. A graduate of Swarthmore College and Cornell University, he has published technical research papers in *Physical Review, Journal of Statistical Physics* and the *Proceedings of the Royal Society*. Styer is an associate editor of the *American Journal of Physics*, and his quantum mechanics software won the 1994 Computers in Physics Educational Software Contest. A man of lively intellect, Styer's goal in life is to keep learning new things, and to that end he invests energy into presenting science to a general audience. 'I learn a lot through research and by teaching technical courses to physics majors,' says Styer, 'but I learn even more by distilling the essence of physics ideas into a rigorously honest yet non-technical presentation for a general audience. To reach this group, I cannot hide my ignorance behind a screen of mathematical formulas or technical jargon.' Professor Styer enjoys running, backpacking, and rearing his two children as well as doing science.

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Dedicated to two extraordinary teachers of quantum mechanics:

John R. Boccio and N. David Mermin

There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.

Hamlet I.v.166

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Preface

This book presents the two central concepts of quantum mechanics in such a way that non-technical readers will learn how to work simple yet meaningful problems, as well as grasp the conceptual bizarreness of the quantal world. Those two central concepts are: (1) The outcome of an experiment cannot, in general, be predicted exactly; only the probabilities of the various outcomes can be found. (2) These probabilities arise through the interference of amplitudes.

The book is based on a short course (only fourteen lectures) that I have presented to general-audience students at Oberlin College since 1989, and thus it is suitable for use as a course textbook. But it is also suitable for individual readers looking for intellectual adventure. The technical background needed to understand the book is limited to high school algebra and geometry. More important prerequisites are an open mind, a willingness to question your ingrained notions, and a spirit of exploration. Like any adventure, reading this book is not easy. But you will find it rewarding as well as challenging, and at the end you will possess a genuine understanding of the subject rather than a superficial gloss.

How can one present a technical subject like quantum mechanics to a non-technical audience? There are several possibilities. One is to emphasize the history of the subject and anecdotes about the founders of the field. Another is to describe the cultural climate, social pressures, and typical working conditions of a quantum physicist today. A third is to describe useful inventions, such as the laser and the transistor, that work through the action of quantum mechanics. A fourth is to outline in general terms the mathematical machinery used by physicists in solving quantum mechanical problems.

I find all four of these approaches unsatisfactory because they emphasize quantum *physicists* rather than quantum *physics*. This book uses instead a fifth approach, which emphasizes how nature behaves rather than how humans behave. Humans have certainly been very clever in discovering and using quantum mechanics, and I am proud of our species for its activities in this regard. But in this book (except for the appendices) the focus rests squarely on nature and not on how we study nature.

In order to solve problems in quantum mechanics, the professional physicist has erected a gigantic and undoubtedly elegant mathematical edifice. This edifice is necessary for finding the answers to specific problems (which is, after all, what physicists are paid to do), but it often conceals rather than reveals the underlying physical principles of quantum mechanics. Physicists, in fact, are often clumsy in their use and understanding of quantum mechanics's central concepts; they are protected from them by a screen of mathematics. (The very name "*quantum* mechanics" memorializes an aspect of atomic physics that is not central to quantum mechanics and that appears in the classical world as well.) This book aims to strip away the machinery of the edifice and bare the raw ideas in their naked form.

An analogy helps to explain this aim. The professional automobile mechanic must be familiar with crankshafts and camshafts, pistons and plugs, transmissions and timing. His familiarity enables him to repair cars and earn his salary. Yet these practical and interesting devices are irrelevant to the central concept of how a car works — which is simply that hot air expands, whence heat from burning gasoline can be converted into motion. Many excellent mechanics are in fact unfamiliar with this central concept. A book on the fundamental workings of automobiles would discuss heat and motion, but would not tell you how to give your car a tune-up. You should expect analogous discussions here: no more and no less.

Above I have described the direct goals of this book. Two other goals are indirect yet just as important. First, I aim to describe scientific thought — its character, its strengths, its limitations — and to inspire an appreciation for the elegance, economy, and beauty of scientific explanations. Second, I hope to demonstrate the importance and power of reason as a tool for solving problems and probing the unknown. The popular press is fond of misstatements like "the belief in an objective reality, accessible to reason, ... suffered a death blow with the advent of modern physics".* The truth is that quantum mechanics is unfamiliar, non-common-sensical, and weird, but it is perfectly logical and rational. Indeed, in the bizarre world of quantum mechanics, it is logic, and not common sense, that is the only sure guiding light. In today's cultural atmosphere — where in-your-face power play has largely displaced rational debate in the arena of public discourse — this point cannot be overemphasized.

* Sources for direct quotations are gathered in appendix C on page 138.

This book describes quantum mechanics as most physicists understand it today. All scientific knowledge is tentative and the pillars of quantum mechanics are no exception. In addition, the experiments and principles described here are all subject to interpretation. I present the standard interpretation, which is not the only one. (I give only fleeting mention to alternative interpretations and formulations not because they are incorrect or unimportant, but because one must have a firm grasp on the standard interpretation before moving on to the alternatives.)

> *Technical aside:* Sometimes it is useful to make a point that is rather technical and that is not essential for developing the book's argument. Such technical asides are labelled and indented, like this sample.

Producing a completely honest yet non-technical account of quantum mechanics is an audacious enterprise, and while developing this treatment I have reached out for help from many people. I need to thank first the 985 Oberlin College students who have, since 1989, taken the course which led to this book. Their questions, objections, doubts, excitement, enthusiasm, and triumphs have inspired many changes — improvements, I hope — in the content and presentation given here, as well as in my own understanding of quantum mechanics. In the spring of 1996 I served as associate instructor for the computer conference course "Demystifying Quantum Mechanics", developed and taught by Edwin F. Taylor. Working with Professor Taylor and the fifteen intrepid students in that class (mostly high school teachers scattered across the nation) was a pleasure that further refined my understanding and this book's presentation.

I received helpful direct comments on this treatment from many of the students mentioned above, and from Gary E. Bowman, Amy Bug, Peter Collings, Rufus Neal, Joe Palmieri, Robert Romer, Dan Sulke, Edwin F. Taylor, and four anonymous reviewers. This is not to say that all of these readers approve of everything I say here — indeed, I know that some of them disagree with me on important points — but I appreciate the contribution that each one of them has made to this work. The illustrations were skillfully drawn by Byron Fouts.

The development of the course which led to this book was supported by a grant from the Sloan Foundation. This acknowledgement may sound like the bland gratitude of someone merely content to receive Sloan's money, but it is not. The encouragement of the foundation, and in particular of program officer Samuel Goldberg, led me to delve deeply into quantum mechanics as a set of physical ideas rather than as an elaborate and somewhat mystical algorithm for solving problems in atomic physics. I have learned much in preparing this account, and

I thank the Sloan Foundation for suggesting that someone other than myself would be interested in what I learned.

I invite you to join the community that has developed this approach and this book. If you have access to the Internet you can send me computer mail at address

Dan.Styer@oberlin.edu

and you will find a World Wide Web page devoted to this book at

http://www.oberlin.edu/physics/dstyer/StrangeQM/

Comments on paper are just as welcome, and should be addressed to

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I offer you my welcome and my best wishes. Enjoy!

1
Introduction

1.1 Capsule history of quantum mechanics

Starting in the seventeenth century, and continuing to the present day, physicists developed a body of ideas that describe much about the world around us: the motion of a cannonball, the orbit of a planet, the working of an engine, the crack of a baseball bat. This body of ideas is called *classical mechanics.*

In 1905, Albert Einstein realized that these ideas didn't apply to objects moving at high speeds (that is, at speeds near the speed of light) and he developed an alternative body of ideas called *relativistic mechanics.* Classical mechanics is wrong in principle, but it is a good approximation to relativistic mechanics when applied to objects moving at low speeds.

At about the same time, several experiments led physicists to realize that the classical ideas also didn't apply to very small objects, such as atoms. Over the period 1900–1927 a number of physicists (Planck, Bohr, Einstein, Heisenberg, de Broglie, Schrödinger, and others) developed an alternative *quantum mechanics.* Classical mechanics is wrong in principle, but it is a good approximation to quantum mechanics when applied to large objects.

1.2 What is the nature of quantum mechanics?

I'm not going to spend any time on the history of quantum mechanics, which is convoluted and fascinating. Instead, I will focus on the ideas developed at the end. What sort of ideas required twenty-eight years of development from this stellar group of scientists?

Einstein's theory of relativity is often (and correctly) described as strange and counterintuitive. Yet, according to a widely used graduate level text,

> [the theory of relativity] is a modification of the structure of mechanics which must not be confused with the far more violent recasting required by quantum theory.

Murray Gell-Mann, probably the most prominent living practitioner of the field, said of quantum mechanics that

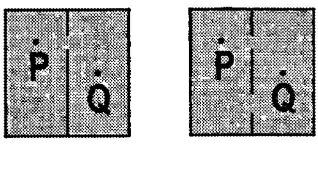
> Nobody feels perfectly comfortable with it.

And the inimitable Richard Feynman, who developed many of the ideas that will be expounded in this book, remarked that

> I can safely say that nobody understands quantum mechanics.

One strange aspect of quantum mechanics concerns predictability. Classical mechanics is *deterministic* — that is, if you know exactly the situation as it is now, then you can predict exactly what it will be at any moment in the future. Chance plays no role in classical mechanics. Of course, it might happen that the prediction is very difficult to perform, or it might happen that it is very difficult to find exactly the current situation, so such a prediction might not be a practical possibility. (This is the case when you flip a coin.) But in principle any such barriers can be surmounted by sufficient work and care. Relativistic mechanics is also deterministic. In contrast, quantum mechanics is *probabilistic* — that is, even in the presence of exact knowledge of the current situation, it is impossible to predict its future exactly, regardless of how much work and care one invests in such a prediction.

Even stranger, however, is quantum mechanical *interference*. I cannot describe this phenomenon in a single paragraph — that is a major job of this entire book — but I can give an example. Suppose a box is divided in half by a barrier with a hole drilled through it, and suppose an atom moves from point P in one half of the box to point Q in the other half. Now suppose a second hole is drilled through the barrier and then the experiment is repeated. The second hole increases the number of possible ways to move from P to Q, so it is natural to guess that its presence will increase the probability of making this move. But in fact — and in accord with the predictions of quantum mechanics — a second hole drilled at certain locations will *decrease* that probability.



The fact that quantum mechanics is strange does not mean that quantum mechanics is unsuccessful. On the contrary, quantum mechanics is the most successful theory that humanity has ever developed; the brightest jewel in our intellectual crown. Quantum mechanics underlies our understanding of atoms, molecules, solids, and nuclei. It is vital for explaining aspects of stellar evolution, chemical reactions, and the interaction of light with matter. It underlies the operation of lasers, transistors, magnets, and superconductors. I could cite reams of evidence backing up these assertions, but I will content myself by describing a single measurement. One electron will be stripped away from a helium atom that is exposed to ultraviolet light below a certain wavelength. This threshold wavelength can be determined experimentally to very high accuracy: it is $50.425\,929\,9 \pm 0.000\,000\,4$ nanometers. The threshold wavelength can also be calculated from quantum mechanics: this prediction is $50.425\,931\,0 \pm 0.000\,002\,0$ nanometers. The agreement between observation and quantum mechanics is extraordinary. If you were to predict the distance from New York to Los Angeles with this accuracy, your prediction would be correct to within the width of your hand. In contrast, classical mechanics predicts that *any* wavelength of light will strip away an electron, that is, that there will be no threshold at all.

1.3 How small is small?

I said above that the results predicted by quantum mechanics differed from the results predicted by classical mechanics only when these ideas were applied to "very small objects, such as atoms". How small is an atom? Cells are small: a typical adult contains about 60 trillion cells. But atoms are far smaller: a typical cell contains about 120 trillion atoms. An atom is twice as small, relative to a cell, as a cell is small, relative to a person. In this book, when I say "small" I mean "*very* small". You've never handled objects this small; I've never handled objects this small; none of your friends has ever handled objects this small. They are completely outside the domain of our common experience. As you read this book, you will find that quantum mechanics is contrary to common sense. There is nothing wrong with this. Common sense applies to commonly encountered situations, and we do not commonly encounter the atomic world.

1.4 The role of mathematics in quantum mechanics

One frequently hears statements to the effect that the ideas of quantum mechanics are highly mathematical and can only be understood through the use of complex mathematics (partial differential equations, Fourier transforms, eigenfunction expansions, etc.).

> One can popularize the quantum theory [only] at the price of gross oversimplification and distortion, ending up with an uneasy compromise between what the facts dictate and what it is possible to convey in ordinary language.

It is certainly true that the professional physicist needs a vast mathematical apparatus in order to solve efficiently the problems of quantum mechanics. (For example, the calculation of the helium stripping threshold wavelength described above was a mathematical *tour de force*.) But this is not, I believe,* because quantum mechanics itself is fundamentally difficult or mathematical. I believe instead that the root rules of quantum mechanics are in fact quite simple. (They are unfamiliar and unexpected, but nevertheless simple.) When these rules are applied to particular situations, they are used over and over again and therefore the *applications* are complicated. An analogy helps explain this distinction. The rules of chess are very simple: they can be written on a single page of paper. But when these rules are applied to particular situations they are used over and over again and result in a complicated game: the applications of the chess rules fill a library.

Indeed, can any fundamental theory be highly mathematical? Electrons know how to obey quantum mechanics, and electrons can neither add nor subtract, much less solve partial differential equations! If something as simple-minded as an electron can understand quantum mechanics, then certainly something as wonderfully complex as the reader of this book can understand it too.

* Not everyone agrees with me.