Other worlds

The volume in position space is V^N.

The volume in momentum space (of dimension dN) is

$$\frac{\pi^{dN/2}}{(dN/2)!}\left[(2m(E+\Delta E))^{dN/2}-(2mE)^{dN/2}\right].$$

So the volume in phase space is

$$\frac{\pi^{dN/2}}{(dN/2)!}(2mE)^{dN/2}V^N\left[\left(1+\frac{\Delta E}{E}\right)^{dN/2}-1\right].$$

and

$$\frac{S}{k_B}=\ln\frac{\text{volume}}{N!h_0^{dN}}=\ln\left\{\left(\frac{2\pi mEV^{2/d}}{h_0^2}\right)^{dN/2}\frac{1}{N!\,(dN/2)!}\left[\left(1+\frac{\Delta E}{E}\right)^{dN/2}-1\right]\right\}$$

To prepare for the thermodynamic limit, define

$$e\equiv\frac{E}{N}, v\equiv\frac{V}{N},\text{ and }\delta\equiv\frac{\Delta E}{N},$$

and use

$$\ln N!\approx N\ln N-N.$$

Together these give

$$\frac{S}{k_B}=\frac{dN}{2}\ln\left(\frac{2\pi ev^{2/d}N^{(1+(2/d))}}{h_0^2}\right)-N\ln N+N-\frac{dN}{2}\ln\frac{dN}{2}+\frac{dN}{2}+\ln\left[\left(1+\frac{\delta}{e}\right)^{dN/2}-1\right].$$

For large values of N, the rightmost expression becomes

$$\ln\left[\left(1+\frac{\delta}{e}\right)^{dN/2}-1\right]\to\frac{dN}{2}\ln\left(1+\frac{\delta}{e}\right),$$

and the entropy per number approaches

$$\frac{S}{k_BN}=\frac{d}{2}\ln\left(\frac{2\pi ev^{2/d}}{h_0^2}\right)+\frac{d}{2}\left(1+\frac{2}{d}\right)\ln N-\ln N+1-\frac{d}{2}\ln\left(\frac{dN}{2}\right)+\frac{d}{2}+\frac{d}{2}\ln\left(1+\frac{\delta}{e}\right).$$

The right hand side contains only three N-dependent expressions. . . and they sum to an expression independent of N! The rightmost term vanishes in the limit $\delta\to 0$, and we produce

$$\begin{aligned}\frac{S}{k_BN}&=\frac{d}{2}\ln\left(\frac{2\pi ev^{2/d}}{h_0^2}\right)+1-\frac{d}{2}\ln\left(\frac{d}{2}\right)+\frac{d}{2}\\&=\frac{d}{2}\ln\left(\frac{4\pi ev^{2/d}}{dh_0^2}\right)+\frac{d+2}{2}\end{aligned}$$

or

$$S(E,V,N)=k_BN\left[\frac{d}{2}\ln\left(\frac{4\pi mEV^{2/d}}{dh_0^2N^{(d+2)/d}}\right)+\frac{d+2}{2}\right].$$