Magnetic systems

$E(S, H)$

$$dE = T\,dS - M\,dH$$

$\Downarrow$ Legendre transformation

$F(T, H) = E - TS$

$$dF = -S\,dT - M\,dH \quad \overset{\text{Maxwell relation}}{\Longrightarrow} \quad \left.\frac{\partial S}{\partial H}\right)_T = \left.\frac{\partial M}{\partial T}\right)_H$$

$\Downarrow$ Legendre transformation

$A(T, M) = F + MH$

$$dA = -S\,dT + H\,dM \quad \overset{\text{Maxwell relation}}{\Longrightarrow} \quad -\left.\frac{\partial S}{\partial M}\right)_T = \left.\frac{\partial H}{\partial T}\right)_M$$

Note particularly the physical/experimental meaning of the first Maxwell relation: To find the quantity on the left, you would measure the heat absorbed by the system as the applied magnetic field is changed while in a thermostatic bath: a very "heat-like" measurement. To find the quantity on the right, you would measure the magnetization of the sample using the same technique that you would use to measure the magnetization of a scout compass needle: a very "E&M-like" measurement. It's not immediately clear that the two quantities even have the same dimensions! Yet these two very different measurements are guaranteed to give always the same result.