Lost in space

Question: A collection of N asteroids floats in space far from other gravitating bodies. Model each asteroid as a uniform hard sphere of radius R and mass m. What quantities are required for a microscopic description of this system? For a macroscopic description?

A **microscopic** description consists of two pieces:

- The mechanical parameters N, m, R, plus Newton's law of gravity — these are the things we would need to know in order to write down the Hamiltonian.
- The dynamical variables $\vec{r}_1$, $\vec{r}_2$, $\ldots \vec{r}_N$ and $\vec{p}_1$, $\vec{p}_2$, $\ldots \vec{p}_N$ — these are the things we would solve for if this were a classical mechanics problem. In addition, because these are spheres and not point particles, the dynamical variables will include, for each sphere, the angular momentum of the sphere about its center of mass: $\vec{\ell}_1$, $\vec{\ell}_2$, $\ldots \vec{\ell}_N$. (If the shapes were, say, rectangular solids rather than spheres, we would need to give the orientation of each asteroid as well. But this is not needed for spheres.)

A **macroscopic** description also consists of two pieces:

- The same mechanical parameters as before.
- The conserved quantities E (energy), $\vec{P}$ (total momentum), and $\vec{L}$ (total angular momentum). (For a box of gas molecules the external wall forces mean that $\vec{P}$ and $\vec{L}$ are not conserved, so they don't become part of the macroscopic description. But for the asteroid case they *are* conserved and hence *are* part of the macroscopic description.)

In the microscopic description $9N$ dynamical variables are specified, whereas in the macroscopic description 7 conserved quantities are specified.