Heat capacity at constant pressure

The argument for C_V was: By definition,

$$C_V(T,V,N) = T\,\frac{\partial S}{\partial T}\bigg)_{V,N}.$$

But the energy differential is

$$dE = T\,dS - p\,dV + \mu\,dN.$$

To reflect the constant V and N in the definition of C_V above, restrict this equation for dE to changes at constant V and N, giving

$$dE = T\,dS \qquad \text{for } V,\ N \text{ constant.}$$

Divide by "dT" to obtain

$$\frac{\partial E}{\partial T}\bigg)_{V,N} = T\,\frac{\partial S}{\partial T}\bigg)_{V,N} = C_V(T,V,N).$$

So the parallel argument for C_p is: By definition,

$$C_p(T,p,N) = T\,\frac{\partial S}{\partial T}\bigg)_{p,N}.$$

But the enthalpy differential is

$$dH = T\,dS + V\,dp + \mu\,dN.$$

To reflect the constant p and N in the definition of C_p above, restrict this equation for dH to changes at constant p and N, giving

$$dH = T\,dS \qquad \text{for } p,\ N \text{ constant.}$$

Divide by "dT" to obtain

$$\frac{\partial H}{\partial T}\bigg)_{p,N} = T\,\frac{\partial S}{\partial T}\bigg)_{p,N} = C_p(T,p,N).$$