Heat capacities in a magnetic system

The reasoning here parallels the reasoning connecting C_V with C_p in fluid systems. There are three main parts:

A: Begin with the known master relation for $E(S, H)$:

$$dE = T\,dS - M\,dH.$$

Apply a Legendre transformation to variables T and H (trade in an S for a T):

$$F = E - TS \qquad \text{so} \qquad dF = -S\,dT - M\,dH.$$

The resulting Maxwell relation is

$$\left.\frac{\partial S}{\partial H}\right)_T = \left.\frac{\partial M}{\partial T}\right)_H \equiv \beta.$$

B: Start with the purely mathematical relation

$$\begin{aligned} dM &= \left.\frac{\partial M}{\partial T}\right)_H dT + \left.\frac{\partial M}{\partial H}\right)_T dH \\ &= \beta\,dT + \chi_T\,dH. \end{aligned}$$

This relation is good for "any sufficiently small change". Restrict it to changes at constant M, so $dM = 0$:

$$-\beta\,dT = \chi_T\,dH$$

whence

$$-\frac{\beta}{\chi_T} = \left.\frac{\partial H}{\partial T}\right)_M.$$

C: Start with the purely mathematical relation

$$\begin{aligned} dS &= \left.\frac{\partial S}{\partial T}\right)_H dT + \left.\frac{\partial S}{\partial H}\right)_T dH \\ &= \frac{C_H}{T}\,dT + \beta\,dH, \end{aligned}$$

where in the last line we used the definition of C_H and the result of part **A**. This relation is good for "any sufficiently small change". Restrict it to changes at constant M, and then divide by dT:

$$\left.\frac{\partial S}{\partial T}\right)_M = \frac{C_H}{T} + \beta\left.\frac{\partial H}{\partial T}\right)_M.$$

Now use the definition of C_M and the result of part **B**:

$$\frac{C_M}{T} = \frac{C_H}{T} + \beta\left(-\frac{\beta}{\chi_T}\right)$$

or

$$C_M = C_H - \frac{T\beta^2}{\chi_T}.$$