Heat capacities for the ideal gas

From the Sackur-Tetrode formula,

$$S(E,V,N) = k_B N \left[\tfrac{3}{2} \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2 N^{5/3}} \right) + \tfrac{5}{2} \right],$$

we have already derived that

$$E = \tfrac{3}{2} N k_B T.$$

Plugging in, this shows that

$$S(T,V,N) = k_B N \left[\tfrac{3}{2} \ln \left(\frac{2\pi m k_B T V^{2/3}}{h_0^2 N^{2/3}} \right) + \tfrac{5}{2} \right].$$

Hence

$$\begin{aligned} S(T,V,N) &= k_B N \tfrac{3}{2} \ln T + \text{stuff independent of } T \\ C_V(T,V,N) &= T \left.\frac{\partial S}{\partial T}\right)_{V,N} = \tfrac{3}{2} k_B N. \end{aligned}$$

We have also derived from the Sackur-Tetrode formula that

$$V = N k_B T / p,$$

so

$$S(T,p,N) = k_B N \left[\tfrac{3}{2} \ln \left(\frac{2\pi m k_B^{5/3} T^{5/3}}{h_0^2 p^{2/3}} \right) + \tfrac{5}{2} \right].$$

Hence

$$\begin{aligned} S(T,p,N) &= k_B N \tfrac{3}{2} \ln T^{5/3} + \text{stuff independent of } T \\ &= k_B N \tfrac{5}{2} \ln T + \text{stuff independent of } T \\ C_p(T,p,N) &= T \left.\frac{\partial S}{\partial T}\right)_{p,N} = \tfrac{5}{2} k_B N. \end{aligned}$$