Chemical potential of an ideal gas

a. The entropy is

$$S(E,V,N) = k_B N \left[\tfrac{3}{2} \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2 N^{5/3}} \right) + \tfrac{5}{2} \right].$$

To prepare for taking the derivative with respect to N, we write this as

$$S(E,V,N) = k_B N \left[\tfrac{3}{2} \left\{ \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2} \right) - \tfrac{5}{3} \ln N \right\} + \tfrac{5}{2} \right].$$

Then

$$\begin{aligned}
\frac{\mu}{T} = -\frac{\partial S}{\partial N} &= -k_B \left[\tfrac{3}{2} \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2 N^{5/3}} \right) + \tfrac{5}{2} \right] - k_B N \left[\tfrac{3}{2}(-\tfrac{5}{3}) \frac{1}{N} \right] \\
&= -k_B \tfrac{3}{2} \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2 N^{5/3}} \right) \\
\mu &= -k_B T \tfrac{3}{2} \ln \left(\frac{4\pi m E V^{2/3}}{3h_0^2 N^{5/3}} \right)
\end{aligned}$$

or, using $E = \tfrac{3}{2} N k_B T$,

$$\begin{aligned}
\mu &= -k_B T \tfrac{3}{2} \ln \left(\frac{4\pi m}{3h_0^2} (\tfrac{3}{2} k_B T) \left(\frac{V}{N} \right)^{2/3} \right) \\
&= -k_B T \ln \left[\left(\frac{2\pi m k_B T}{h_0^2} \right)^{3/2} \frac{V}{N} \right].
\end{aligned}$$

b. The value (and sign) of μ depends on the value selected for the arbitrary constant h_0. But for any selection of h_0, if T is high enough, and V/N high enough, the $[\sim]$ above becomes greater than 1, so $\ln[\sim]$ is positive and μ is negative.

c. The derivative of chemical potential with respect to temperature is

$$\frac{\partial \mu}{\partial T} = \frac{\mu}{T} - k_B T \frac{\partial \ln[\sim]}{\partial T}$$

but $\ln[\sim] = \ln(\text{stuff independent of } T) + \tfrac{3}{2} \ln T$ so

$$\frac{\partial \mu}{\partial T} = \frac{\mu}{T} - \frac{3}{2} k_B.$$

At low density $\mu < 0$, whence $\dfrac{\partial \mu}{\partial T} < 0$, whence μ decreases with T.