Accessible configurations of a spin system

a. Maximum possible energy is NmH — all spins down.

Minimum possible energy is $-NmH$ — all spins up.

Spacing between energy levels is $2mH$ — the energy to flip one spin.

b. The number of microstates (configurations) consistent with, say, five down spins is

$$\frac{N(N-1)(N-2)(N-3)(N-4)}{5!} = \frac{N!}{5!\,(N-5)!}.$$

In general, the number of microstates consistent with $n_\downarrow$ down spins is the binomial coefficient

$$\frac{N!}{n_\uparrow!\,n_\downarrow!}.$$

These microstates correspond to the energy level with energy

$$E = -(n_\uparrow - n_\downarrow)mH.$$

c. To write this microstate count as a function of macroscopic quantities only, we must solve for $n_\uparrow$ and $n_\downarrow$ in terms of E, N, and H. First write

$$\begin{aligned} n_\uparrow + n_\downarrow &= N \\ n_\uparrow - n_\downarrow &= -E/mH, \end{aligned}$$

and then solve to find

$$\begin{aligned} n_\uparrow &= \tfrac{1}{2}(N - E/mH) \\ n_\downarrow &= \tfrac{1}{2}(N + E/mH). \end{aligned}$$

d. The number of configurations in this energy range is approximately the number of energy levels in the range times the number of configurations in a typical energy level. If $n_\uparrow$ and $n_\downarrow$ represent typical values for a level within this range, then

$$\Omega \approx \left(\frac{N!}{n_\uparrow!\,n_\downarrow!}\right)\left(\frac{\Delta E}{2mH}\right).$$

Expressing this formula using the results of part (c) gives

$$\Omega(E, \Delta E, H, N) \approx \frac{N!}{\left[\frac{1}{2}(N + E/mH)\right]!\,\left[\frac{1}{2}(N - E/mH)\right]!}\;\frac{\Delta E}{2mH}.$$