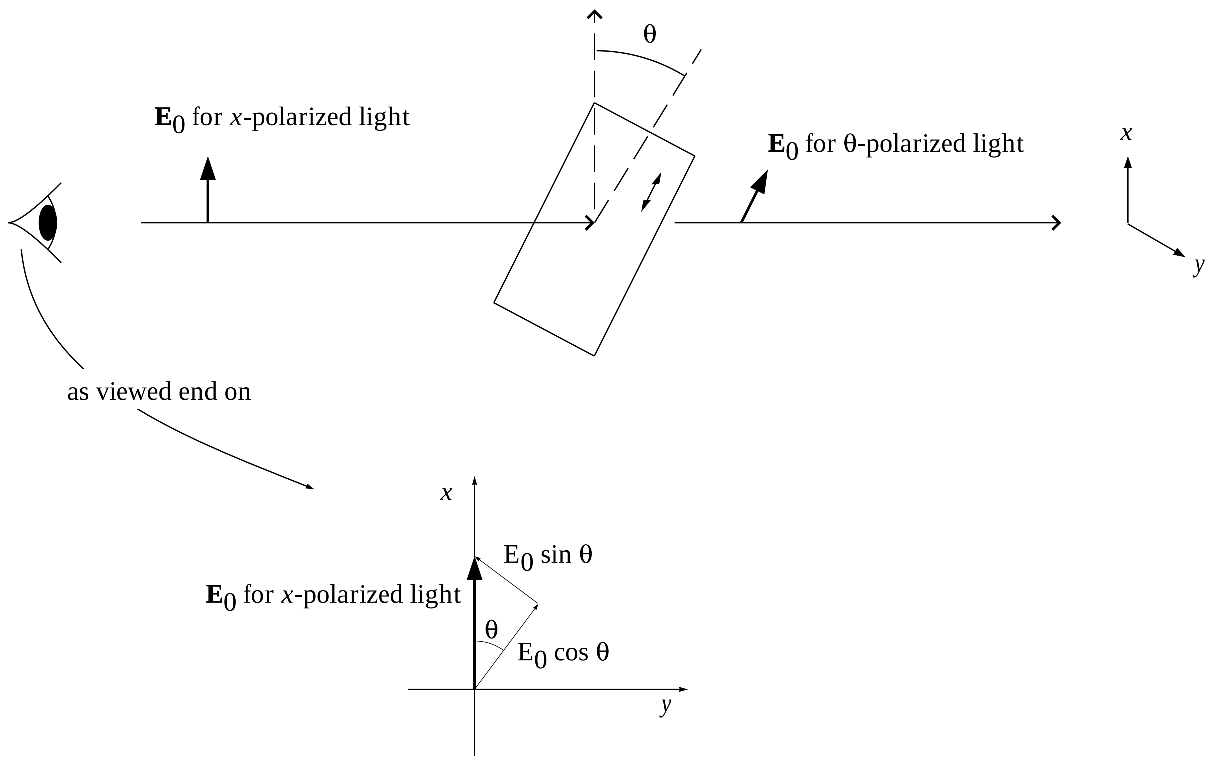
Photon Polarization

1. Classical description of polarized light

Recall that the intensity of a light beam is proportional to the square of its amplitude. That is, if a light beam is $\mathbf{E}(z,t) = \mathbf{E}_0 \cos(kz - \omega t)$, then its amplitude is proportional to $|\mathbf{E}_0|^2$.



The figure makes it clear that as the x-polarized light passes through the polaroid sheet, the component $E_0 \sin\theta$ is erased. The θ-polarized beam has amplitude $E_0 \cos\theta$, so the beam intensity is diminished from I_0 to $I_0 \cos^2\theta$.

⟦*Grading:* 5 points for any sort of diagram or argument; 5 points for result $I_0 \cos^2\theta$.⟧

2. Quantal description of polarized light: Analyzers

$$|\langle x|\theta\rangle|^2 = \cos^2\theta \qquad |\langle x|\theta + 90^\circ\rangle|^2 = \sin^2\theta$$

These analyzer (or "measurement") experiments determine the *magnitude* of each probability amplitude.

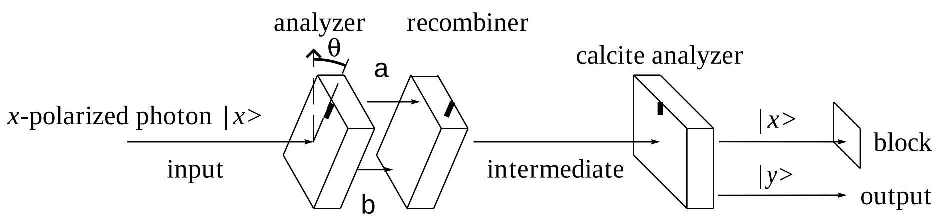
The two states are complete because any incoming photon emerges from either the θ port or the $\theta + 90^\circ$ port.

The two states are orthogonal because when a $|\theta\rangle$ photon encounters a θ-analyzer, it emerges from the θ port with probability one and from the $\theta + 90^\circ$ port with probability zero.

⟦*Grading:* 6 points for the two probabilities; 2 points for completeness; 2 points for orthogonality.⟧

3. Interference

Form a θ analyzer loop by tilting the x, y analyzer loop by the angle θ.



Experiment 1: Slot a blocked.

probability of passing from input to intermediate is $|\langle\theta + 90^\circ|x\rangle|^2 = \sin^2\theta$
state of intermediate photon is $|\theta + 90^\circ\rangle$
probability of passing from intermediate to output is $|\langle y|\theta + 90^\circ\rangle|^2 = \cos^2\theta$
probability of passing from input to output is $\sin^2\theta\cos^2\theta$

Experiment 2: Slot b blocked.

probability of passing from input to intermediate is $|\langle\theta|x\rangle|^2 = \cos^2\theta$
state of intermediate photon is $|\theta\rangle$
probability of passing from intermediate to output is $|\langle y|\theta\rangle|^2 = \sin^2\theta$
probability of passing from input to output is $\cos^2\theta\sin^2\theta$

Experiment 3: Both slots open.

probability of passing from input to intermediate is 1
state of intermediate photon is $|x\rangle$
probability of passing from intermediate to output is $|\langle y|x\rangle|^2 = 0$
probability of passing from input to output is 0

And clearly,

$$0 \neq 2\sin^2\theta\cos^2\theta \quad !$$

An equation representing experiment 3 is:

$$\langle y|\theta\rangle\langle\theta|x\rangle + \langle y|\theta+90^\circ\rangle\langle\theta+90^\circ|x\rangle = \langle y|x\rangle = 0 \tag{1}$$

Problem 2 above gives us the magnitudes

$$|\langle x|\theta\rangle| = |\cos\theta| \qquad |\langle x|\theta+90^\circ\rangle| = |\sin\theta|. \tag{2}$$

And because $|\langle x|\theta\rangle|^2 + |\langle y|\theta\rangle|^2 = 1$; $|\langle x|\theta+90^\circ\rangle|^2 + |\langle y|\theta+90^\circ\rangle|^2 = 1$, we have

$$|\langle y|\theta\rangle| = |\sin\theta| \qquad |\langle y|\theta+90^\circ\rangle| = |\cos\theta|. \tag{3}$$

The easiest way to satisfy equations (1), (2), and (3) simultaneously is to pick all the amplitudes real and one of them negative. The conventional choice is

$$\begin{aligned}
\langle x|\theta\rangle &= \cos\theta \\
\langle y|\theta\rangle &= \sin\theta \\
\langle x|\theta+90^\circ\rangle &= -\sin\theta \\
\langle y|\theta+90^\circ\rangle &= \cos\theta
\end{aligned}$$

⟦A general point on determining probability amplitudes: Analyzer experiments give us the *magnitudes* through equations like (2) and (3), while interference experiments give us the *phases* through equations like (1).⟧

⟦*Grading:* 6 points for a set of experiments showing interference (a single experiment earns only 3 points — interference is manifested through a combination of experiments); 2 points for the interference equation (1); 2 points for the amplitudes.⟧

4. Circular polarization

Can real values

$$\langle R|\ell p\rangle = \pm 1/\sqrt{2} \qquad \langle L|\ell p\rangle = \pm 1/\sqrt{2}$$

satisfy

$$\langle\theta|R\rangle\langle R|x\rangle + \langle\theta|L\rangle\langle L|x\rangle = \langle\theta|x\rangle = \cos\theta \quad ?$$

Certainly not! In any such attempt the left-hand side could take on only three possible values, namely 1, 0, or −1, and the the right-hand side $\cos\theta$ certainly takes on values other than these! However, the complex amplitudes suggested in the question do work, because

$$\begin{aligned} & \langle\theta|R\rangle\langle R|x\rangle + \langle\theta|L\rangle\langle L|x\rangle \\ = & \; (e^{i\theta}/\sqrt{2})(1/\sqrt{2}) + (e^{-i\theta}/\sqrt{2})(1/\sqrt{2}) \\ = & \; \frac{e^{i\theta} + e^{-i\theta}}{2} \\ = & \; \cos\theta. \end{aligned}$$

⟦*Grading:* 5 points for argument that real amplitudes add to 1, 0, or −1; 5 points for checking the proposed (complex) amplitudes.⟧