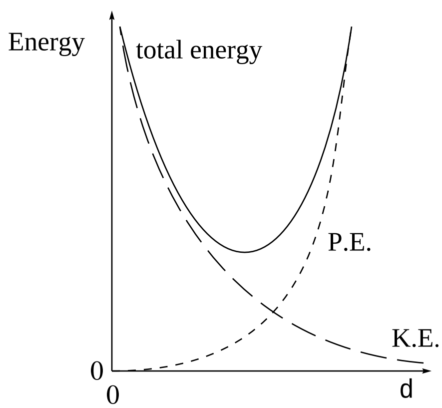
Ground State of the Simple Harmonic Oscillator

From the discussion in the problem statement,

$$\text{P.E.} = \frac{m\omega^2}{8} d^2, \qquad \text{K.E.} = \frac{\hbar^2}{32\,m} \frac{1}{d^2},$$

and the total energy is

$$E(d) = \frac{m\omega^2}{8} d^2 + \frac{\hbar^2}{32\,m} \frac{1}{d^2}.$$



The minimum falls at d_0 where $E'(d) = 0$ so

$$\frac{m\omega^2}{4} d_0 - \frac{\hbar^2}{16\,m} \frac{1}{d_0^3} = 0$$

$$d_0^2 = \frac{\hbar}{2m\omega}$$

whence

$$E(d_0) = \frac{1}{8}\hbar\omega.$$

This estimate for the ground state energy is four times too small, but on the other hand it's considerably easier to find than the true ground state energy!

Note that if $\hbar \to 0$, the P.E. curve does not change, but the K.E. curve moves left and shrinks down into the corner. The P.E. curve dominates, so the minimum energy comes at the minimum P.E., namely $d_0 = 0$.

⟦*Grading:* There are many ways to solve this problem, and they come up with various solutions. In general, 5 points for any discussion about how narrow wavepackets have low PE and high KE, while wide wavepackets have high PE and low KE; and 5 points for turning that argument into a an estimate for the ground state energy that would take the form (dimensionless number)$\hbar\omega$.⟧