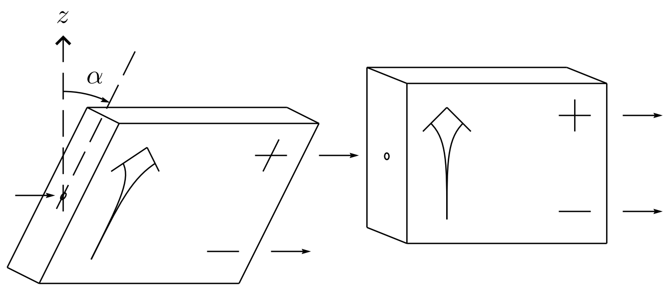
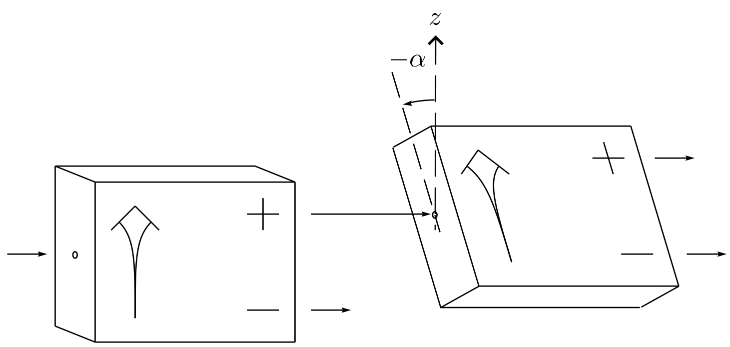
Exit probabilities

(a) Assemblage presented in assignment:



In your mind, rotate the entire assemblage by angle α counterclockwise about the axis pointing right. This will not affect the results.[1] Rotated assemblage:



After rotation the experiment is a vertical analyzer followed by an analyzer rotated from the vertical by $\theta = -\alpha$. (Or you can use $\theta = 360° - \alpha$... it's the same thing.) This is the situation of section 2.2.7. The probability of exiting from the + port is thus

$$\cos^2(-\alpha/2) = \cos^2(\alpha/2).$$

Or, if you had used $\theta = 360° - \alpha$, you would have found

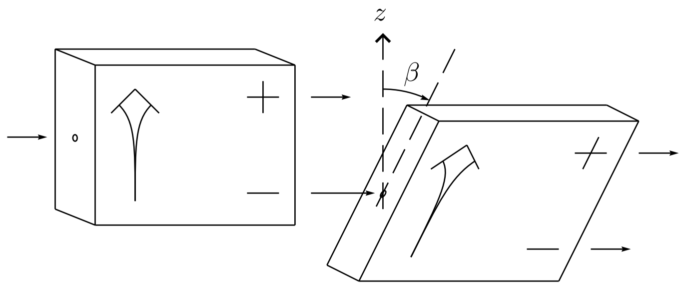
$$\cos^2((360° - \alpha)/2) = [\cos(180° - \alpha/2)]^2 = [-\cos(\alpha/2)]^2 = \cos^2(\alpha/2).$$

[As a check, note that in the special case $\alpha = 0$ this expression gives the correct probability of 1. In the special case $\alpha = 180°$, it gives the correct probability of 0. In the special case $\alpha = 90°$ (discussed in section 2.2.5), it gives the correct probability of $\frac{1}{2}$.] The probability of exiting from the − port is

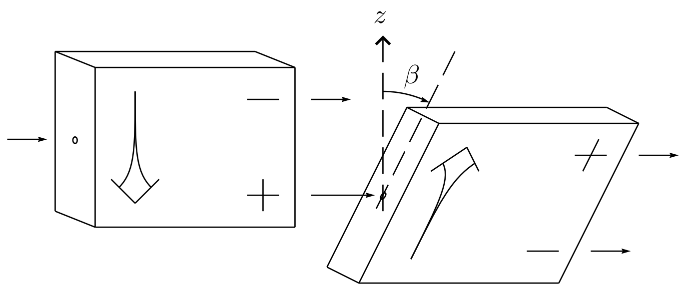
$$1 - \cos^2(\alpha/2) = \sin^2(\alpha/2).$$

[1] Alternatively and equivalently, peer down the apparatus to the right, then rotate your head *clockwise* by angle α. You *certainly* cannot affect the results by tilting your head!

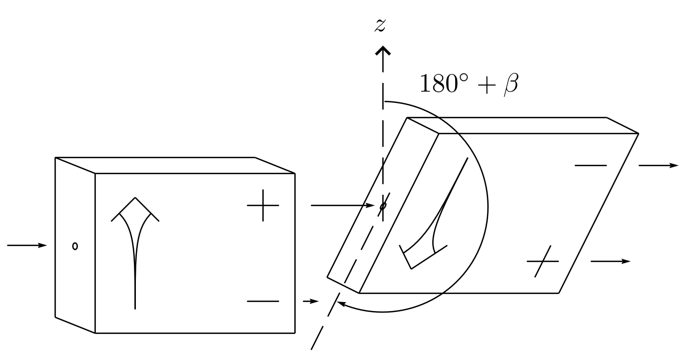
(b) Assemblage presented in assignment:



This experiment is equivalent to an upside-down analyzer, with the atom emerging from its + port fed into a β-analyzer:



In your mind, rotate the entire assemblage by 180° giving:



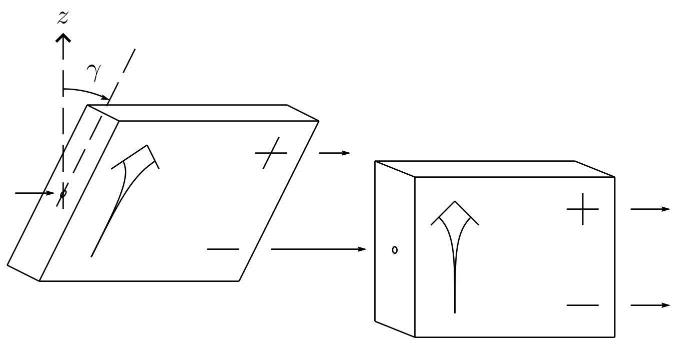
Now the experiment is a vertical analyzer followed by an analyzer rotated $\theta = 180^\circ + \beta$ relative to the vertical. The probability of exiting from the + port is

$$\cos^2((180^\circ + \beta)/2) = [\cos(90^\circ + \beta/2)]^2 = [-\sin(\beta/2)]^2 = \sin^2(\beta/2).$$

The probability of exiting from the − port is

$$1 - \sin^2(\beta/2) = \cos^2(\beta/2).$$

(c) Assemblage presented in assignment:



In your mind, rotate the entire assemblage by angle γ counterclockwise about the axis pointing right. Now it is the experiment of part (b), with $\beta = -\gamma$.

From the result of part (b), the probability of exiting from the + port is

$$\sin^2(\beta/2) = \sin^2(-\gamma/2) = \sin^2(\gamma/2).$$

The probability of exiting from the − port is

$$1 - \sin^2(\gamma/2) = \cos^2(\gamma/2).$$

⟦*Grading:* 1 point for free, 3 points for each of parts (a), (b), and (c). Within each part there's 1 point for the argument about "rotating the entire assemblage", 1 point for finding the probability of exiting from the + port, and 1 point for finding the probability of exiting from the − port. In these model solutions, I present the "argument about 'rotating the entire assemblage'" in great detail, with lots of sketches. Students don't need to make the argument nearly so detailed, but they need to make some sort of argument.⟧