Quantum Mechanics

Sample Exam for Final Examination

1 A one-particle system is in angular momentum eigenstate $|j, m\rangle$ so that

$$\begin{aligned} \hat{J}^2|j,m\rangle &= \hbar^2 j(j+1)|j,m\rangle \\ \hat{J}_z|j,m\rangle &= \hbar m|j,m\rangle. \end{aligned}$$

What is the expectation value and uncertainty of $\hat{J}_x$ in this state?

2 For a hydrogen atom in the energy eigenstate with $n = 4$, $\ell = 3$, and $m = 0$, the mean separation between nucleus and electron is 0.952 nm. Find the mean separation between nucleus and electron for a singly ionized helium ion in its energy eigenstate with $n = 4$, $\ell = 3$, and $m = 0$. (Ignore nuclear motion.)

3 A spin-$\frac{1}{2}$ particle in initial state $|\psi(0)\rangle$ is equally likely to be found in either of the basis states $|\uparrow\rangle$ or $|\downarrow\rangle$.

a. Write a normalized representation of $|\psi(0)\rangle$ in the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$.

b. If the Hamiltonian is represented, in the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$, by the matrix

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

then find the representation of the state $|\psi(t)\rangle$ for all time.

4 Prove the quantum mechanical virial theorem for a one-dimensional system by following these steps:

a. For a system in an energy eigenstate, what is $\frac{d}{dt}\langle\hat{x}\hat{p}\rangle$?

b. Relate $\frac{d}{dt}\langle\hat{x}\hat{p}\rangle$ to $[\hat{x}\hat{p}, \hat{H}]$, where $\hat{H}$ is the Hamiltonian $\frac{\hat{p}^2}{2m} + V(\hat{x})$.

c. Evaluate the commutator to show that, for an energy eigenstate,

$$2\langle\widehat{\mathrm{KE}}\rangle = -\langle\hat{x}F(\hat{x})\rangle \quad \text{where} \quad F(x) = -\frac{\partial V(x)}{\partial x}.$$

What materials (books, notes, web sites, etc.) did you consult while taking this exam?