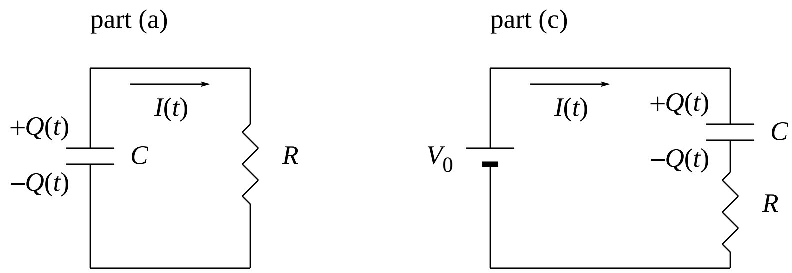
Energy in capacitor discharge

Griffiths, *Electrodynamics*, fourth edition, problem 7.2



a. Initial charge is $Q_0 = V_0 C$.

Integrate $\int \vec{E} \cdot d\vec{\ell}$ around loop, giving

$$\frac{Q(t)}{C} - I(t)R = 0.$$

But

$$I(t) = -\frac{dQ(t)}{dt}$$

so

$$\frac{dQ(t)}{dt} + \frac{1}{RC}Q(t) = 0.$$

Solution:

$$\begin{aligned} Q(t) &= Q_0 e^{-t/RC}, \\ I(t) &= \frac{Q_0}{RC} e^{-t/RC}. \end{aligned}$$

b. Initial potential energy is $\frac{1}{2}CV_0^2 = \frac{1}{2}\frac{Q_0^2}{C}$.

Heat dissipated at resistor is

$$\begin{aligned} \int_0^\infty I^2(t)R\,dt &= \left(\frac{Q_0}{RC}\right)^2 R \int_0^\infty e^{-2t/RC}\,dt = \left(\frac{Q_0}{RC}\right)^2 R \left[-\frac{RC}{2}e^{-2t/RC}\right]_0^\infty \\ &= -\frac{1}{2}\frac{Q_0^2}{C}\left[e^{-2t/RC}\right]_0^\infty = -\frac{1}{2}\frac{Q_0^2}{C}[0-1] = \frac{1}{2}\frac{Q_0^2}{C}. \end{aligned}$$

c. Integrate $\int \vec{E} \cdot d\vec{\ell}$ around loop, giving

$$V_0 - \frac{Q(t)}{C} - I(t)R = 0.$$

But

$$I(t) = +\frac{dQ(t)}{dt}$$

so

$$\frac{dQ(t)}{dt} + \frac{1}{RC}Q(t) = \frac{V_0}{R}.$$

Solution:

$$\begin{aligned} Q(t) &= V_0 C \left(1 - e^{-t/RC}\right), \\ I(t) &= \frac{V_0}{R} e^{-t/RC}. \end{aligned}$$

d. Energy put into circuit by battery is

$$\int_0^\infty I(t) V_0 \, dt = \frac{V_0^2}{R} \int_0^\infty e^{-t/RC} \, dt = \frac{V_0^2}{R} \left[-RCe^{-t/RC}\right]_0^\infty = CV_0^2.$$

Heat dissipated at resistor is

$$\begin{aligned} \int_0^\infty I^2(t) R \, dt &= \left(\frac{V_0}{R}\right)^2 R \int_0^\infty e^{-2t/RC} \, dt = \frac{V_0^2}{R} \left[-\frac{RC}{2} e^{-2t/RC}\right]_0^\infty \\ &= -\frac{1}{2} CV_0^2 \left[e^{-2t/RC}\right]_0^\infty = \frac{1}{2} CV_0^2. \end{aligned}$$

Energy stored in capacitor is

$$\frac{1}{2} CV_0^2.$$

What words describe this result? Satisfaction, bliss, rapture … equipartition?