Building basis states

Suppose you had three particles and three "building block" levels (say the orthonormal levels $\eta_1(x)$, $\eta_3(x)$, and $\eta_7(x)$). Construct and count the possible three-particle states representing (a) three non-identical particles; (b) three identical bosons; and (c) three identical fermions.

For three non-identical particles, the $3^3 = 27$ states are [[2 points]]

$\eta_1(x_A)\eta_1(x_B)\eta_1(x_C)$
$\eta_1(x_A)\eta_1(x_B)\eta_3(x_C)$
$\eta_1(x_A)\eta_1(x_B)\eta_7(x_C)$
$\eta_1(x_A)\eta_3(x_B)\eta_1(x_C)$
$\eta_1(x_A)\eta_3(x_B)\eta_3(x_C)$
$\eta_1(x_A)\eta_3(x_B)\eta_7(x_C)$
$\eta_1(x_A)\eta_7(x_B)\eta_1(x_C)$
$\eta_1(x_A)\eta_7(x_B)\eta_3(x_C)$
$\eta_1(x_A)\eta_7(x_B)\eta_7(x_C)$
$\eta_3(x_A)\eta_1(x_B)\eta_1(x_C)$
$\eta_3(x_A)\eta_1(x_B)\eta_3(x_C)$
$\eta_3(x_A)\eta_1(x_B)\eta_7(x_C)$
$\eta_3(x_A)\eta_3(x_B)\eta_1(x_C)$
$\eta_3(x_A)\eta_3(x_B)\eta_3(x_C)$
$\eta_3(x_A)\eta_3(x_B)\eta_7(x_C)$
$\eta_3(x_A)\eta_7(x_B)\eta_1(x_C)$
$\eta_3(x_A)\eta_7(x_B)\eta_3(x_C)$
$\eta_3(x_A)\eta_7(x_B)\eta_7(x_C)$
$\eta_7(x_A)\eta_1(x_B)\eta_1(x_C)$
$\eta_7(x_A)\eta_1(x_B)\eta_3(x_C)$
$\eta_7(x_A)\eta_1(x_B)\eta_7(x_C)$
$\eta_7(x_A)\eta_3(x_B)\eta_1(x_C)$
$\eta_7(x_A)\eta_3(x_B)\eta_3(x_C)$
$\eta_7(x_A)\eta_3(x_B)\eta_7(x_C)$
$\eta_7(x_A)\eta_7(x_B)\eta_1(x_C)$
$\eta_7(x_A)\eta_7(x_B)\eta_3(x_C)$
$\eta_7(x_A)\eta_7(x_B)\eta_7(x_C)$

For three identical bosons, the 10 states are ⟦3 points⟧

$$\eta_1(x_A)\eta_1(x_B)\eta_1(x_C)$$
$$\frac{1}{\sqrt{3}}[\eta_1(x_A)\eta_1(x_B)\eta_3(x_C) + \eta_1(x_A)\eta_3(x_B)\eta_1(x_C) + \eta_3(x_A)\eta_1(x_B)\eta_1(x_C)]$$
$$\frac{1}{\sqrt{3}}[\eta_1(x_A)\eta_1(x_B)\eta_7(x_C) + \eta_1(x_A)\eta_7(x_B)\eta_1(x_C) + \eta_7(x_A)\eta_1(x_B)\eta_1(x_C)]$$
$$\frac{1}{\sqrt{3}}[\eta_1(x_A)\eta_3(x_B)\eta_3(x_C) + \eta_3(x_A)\eta_1(x_B)\eta_3(x_C) + \eta_3(x_A)\eta_3(x_B)\eta_1(x_C)]$$
$$\frac{1}{\sqrt{6}}[\eta_1(x_A)\eta_3(x_B)\eta_7(x_C) + \eta_1(x_A)\eta_7(x_B)\eta_3(x_C) + \eta_7(x_A)\eta_1(x_B)\eta_3(x_C) + \eta_7(x_A)\eta_3(x_B)\eta_1(x_C) + \eta_3(x_A)\eta_7(x_B)\eta_1(x_C) + \eta_3(x_A)\eta_1(x_B)\eta_7(x_C)]$$
$$\frac{1}{\sqrt{3}}[\eta_1(x_A)\eta_7(x_B)\eta_7(x_C) + \eta_7(x_A)\eta_1(x_B)\eta_7(x_C) + \eta_7(x_A)\eta_7(x_B)\eta_1(x_C)]$$
$$\eta_3(x_A)\eta_3(x_B)\eta_3(x_C)$$
$$\frac{1}{\sqrt{3}}[\eta_3(x_A)\eta_3(x_B)\eta_7(x_C) + \eta_3(x_A)\eta_7(x_B)\eta_3(x_C) + \eta_7(x_A)\eta_3(x_B)\eta_3(x_C)]$$
$$\frac{1}{\sqrt{3}}[\eta_3(x_A)\eta_7(x_B)\eta_7(x_C) + \eta_7(x_A)\eta_3(x_B)\eta_7(x_C) + \eta_7(x_A)\eta_7(x_B)\eta_3(x_C)]$$
$$\eta_7(x_A)\eta_7(x_B)\eta_7(x_C)$$

For three identical fermions, the 1 state is ⟦3 points⟧

$$\frac{1}{\sqrt{6}}[\eta_1(x_A)\eta_3(x_B)\eta_7(x_C) - \eta_1(x_A)\eta_7(x_B)\eta_3(x_C) + \eta_7(x_A)\eta_1(x_B)\eta_3(x_C) - \eta_7(x_A)\eta_3(x_B)\eta_1(x_C) + \eta_3(x_A)\eta_7(x_B)\eta_1(x_C) - \eta_3(x_A)\eta_1(x_B)\eta_7(x_C)]$$

For three particles with M levels — make counts using "3 balls in M buckets" ideas. ⟦2 points⟧

	number of 3-particle states		
case	general	$M = 3$	$M = 4$
non-identical particles	M^3	27	64
identical bosons	$\frac{M(M+1)(M+2)}{3!}$	10	20
identical fermions	$\frac{M(M-1)(M-2)}{3!}$	1	4