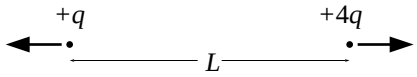
Using Coulomb's law

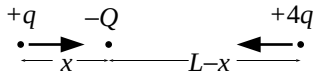
Situation before the third charge is added:



$$\text{The magnitude of each force shown above is} \quad \frac{1}{4\pi\epsilon_0}\frac{4q^2}{L^2}. \tag{1}$$

(a.) To produce equilibrium, the forces due to the third charge must *oppose* the forces shown above. Thus the third charge must be negative and must be located *between* the first two charges.

Forces due to the third charge are:



$$\text{The magnitude of the force on the left is} \quad \frac{1}{4\pi\epsilon_0}\frac{qQ}{x^2}. \tag{2}$$

$$\text{The magnitude of the force on the right is} \quad \frac{1}{4\pi\epsilon_0}\frac{4qQ}{(L-x)^2}. \tag{3}$$

Equilibrium occurs when

$$\frac{qQ}{x^2} = \frac{4q^2}{L^2} \quad \text{and} \quad \frac{4qQ}{(L-x)^2} = \frac{4q^2}{L^2}, \tag{4}$$

that is when

$$QL^2 = 4qx^2 \quad \text{and} \quad QL^2 = q(L-x)^2. \tag{5}$$

Solving these last two simultaniously for x gives

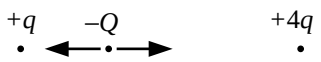
$$4x^2 = (L-x)^2 \quad \Longrightarrow \quad \pm 2x = L - x \quad \Longrightarrow \quad x = -L \text{ or } 3x = L.$$

The solution $x = -L$ is spurious... we've already agreed that the third charge must be *between* the first two. Thus we take $x = \frac{1}{3}L$ and plug that back into equation (5) to find Q. The results are

$$x = \tfrac{1}{3}L \quad \text{and} \quad Q = \tfrac{4}{9}q. \tag{6}$$

(Recall that the third charge is not Q, but $-Q$.)

(b.) Stability analysis. At equilibrium, the forces *on* the third charge are



Suppose the third charge shifts a bit to the left (while the first two charges remain fixed). Then the leftward force increases and the rightward force decreases. Thus a displacement to the left results in a net force to the left. Unstable equilibrium!

⟦You could have also analyzed a shift of the third charge to the right, or up, or down. Or a shift in either of the first two charges. But once you find one instability, you can stop. . . the equilibrium is unstable if any instailities exist.⟧

Grading: 2 points for some sort of startup, usually a graph
1 point for writing forces as in equation (1)
1 point for writing forces as in equation (2)
1 point for writing forces as in equation (3)
2 points for setting forces equal as in equation (4)
2 points for reaching solution (6)
1 point for stability analysis