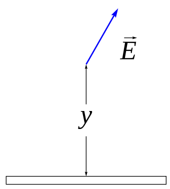
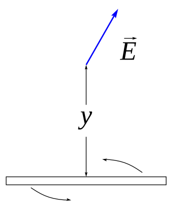
Rod of charge

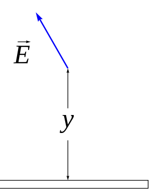
(a.)



Suppose $\vec{E}$ pointed this way.



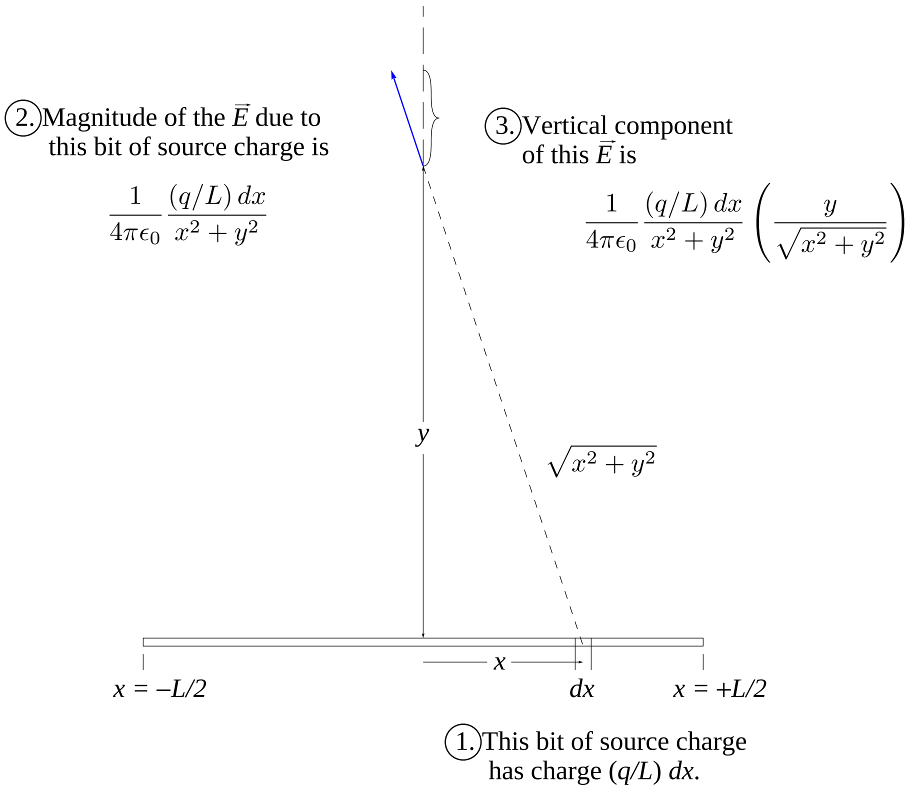
Then if you rotated the rod 180 degrees about a vertical axis . . .



. . . the $\vec{E}$ would rotate right along with the charge.

But after the rotation you're back to exactly the same charge distribution you started with, so you must have the same $\vec{E}$! The only directions that rotate 180° yet end up as they started off are straight up and straight down, so $\vec{E}$ must be one of those.

(b.)



⟦I have defined x with an origin at the center of the rod, not at the left end of the rod, in order to respect the left-right symmetry of the problem. I could have defined x otherwise, and I would have gotten the right answer, but the intermediate steps would have been more complicated.⟧

Thus, the total $\vec{E}$ has magnitude

$$\frac{1}{4\pi\epsilon_0}(q/L)y\int_{-L/2}^{+L/2}\frac{1}{(x^2+y^2)^{3/2}}\,dx.$$

This integral is tabulated (for example Dwight equation 200.03) or you could use a computer algebra system like Mathematica:

$$\int_{-L/2}^{+L/2}\frac{1}{(x^2+y^2)^{3/2}}\,dx=\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_{-L/2}^{+L/2}=\frac{L}{y^2((L/2)^2+y^2)^{1/2}}.$$

Thus the magnitude of the total $\vec{E}$ is

$$\frac{1}{4\pi\epsilon_0}\frac{q}{y((L/2)^2+y^2)^{1/2}}.$$

This is equivalent to equation 5.12 in the textbook LSM.

(c.)

Candidate (1) gives ∞ when $y = L/2$, imaginary numbers when $y < L/2$.
Candidate (2) is dimensionally incorrect.
Candidate (3) is correct.
Candidate (4) gives, when $L = 0$, $E = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{\sqrt{2}\, y^2}$, in violation of Coulomb's law.

(d.)

When y increases, E decreases. Good.
When q increases, E increases. Good.
When L increases, E decreases. Yes... more of the charge is far from the field point, and a lot of the $\vec{E}$ due to the rod tips goes into pointing horizontally and canceling out.

(e.) If $y \gg L$, then $y^2 \gg (L/2)^2$, so $(L/2)^2 + y^2 \approx y^2$.

Thus the magnitude of total $\vec{E}$ is approximately $\dfrac{1}{4\pi\epsilon_0} \dfrac{q}{y^2}$... Coulomb's law!

Grading: 2 points for part **(a.)**
3 points for part **(b.)** [quoting LSM equation 5.12 correctly gives full credit]
2 points for part **(c.)**
2 points for part **(d.)**
1 point for part **(e.)**