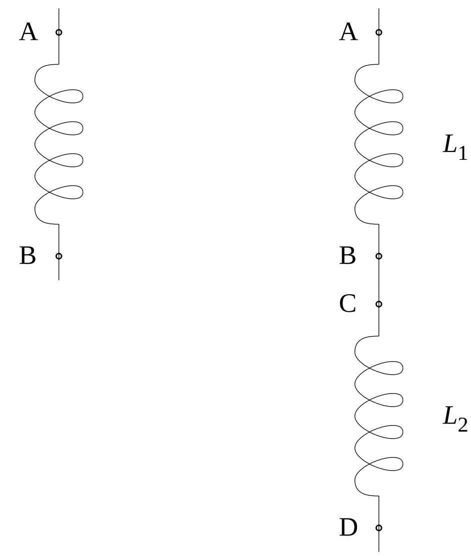
Inductors in series



For a single inductor (see above left)

$$\int_A^B \vec{E} \cdot d\vec{\ell} \equiv \mathcal{E} = -L\frac{di}{dt}.$$

In class we showed that L depends only on the geometry of the inductor by assuming that $\vec{E}$ is caused only by the changing $\vec{B}$ of the inductor. If $\vec{E}$ due to other sources (such as the changing $\vec{B}$ of a different inductor) exists, then this relation is *not* correct.

For two inductors in series (see above right)

$$\begin{aligned}
\mathcal{E}_{\text{total}} &= \int_A^D \vec{E} \cdot d\vec{\ell} &(1)\\
&= \int_A^B \vec{E} \cdot d\vec{\ell} + \int_C^D \vec{E} \cdot d\vec{\ell} &(2)\\
&= -L_1\frac{di}{dt} - L_2\frac{di}{dt} &(3)\\
&= -(L_1 + L_2)\frac{di}{dt} &(4)
\end{aligned}$$

whence

$$L_{eq} = L_1 + L_2.$$

In going from equation (2) to equation (3), we assumed that $\vec{E}$ at inductor 1 is caused only by the $d\vec{B}/dt$ in inductor 1, and not by the $d\vec{B}/dt$ in inductor 2. Similarly for inductor 2. This assumption is correct when the two inductors are far apart.

Grading: There are many ways to solve this problem. Any serious attempt earns full credit (10 points).