

Model Solutions to Second Exam, Makeup Version

1. Power dissipation in a resistor is $P = i\Delta V = (\Delta V)^2/R$. Because ΔV stays constant during the clip out, the desired ratio is

$$\frac{P_{\text{after}}}{P_{\text{before}}} = \frac{R_{\text{before}}}{R_{\text{after}}}.$$

Initially the equivalent resistance is

$$\frac{1}{R_{\text{before}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

and after the clip out the resistance is $R_{\text{after}} = R_1$. Thus

$$\frac{R_{\text{after}}}{R_{\text{before}}} = 1 + \frac{R_1}{R_2} = \frac{R_1 + R_2}{R_2},$$

and

$$\frac{P_{\text{after}}}{P_{\text{before}}} = \frac{R_{\text{before}}}{R_{\text{after}}} = \frac{R_2}{R_1 + R_2} = \frac{15\ \Omega}{34\ \Omega + 15\ \Omega} = 0.31.$$

2. The pick-up coil emf is proportional to the source coil frequency, so it becomes

$$\left(\frac{50\ \text{Hz}}{60\ \text{Hz}}\right) (36\ \text{mV}) = 30\ \text{mV}.$$

3.

$$B(t) = \mu_0 n i(t)$$

$$\Phi_B(t) = \pi r^2 B(t) = \pi r^2 \mu_0 n i(t)$$

$$\begin{aligned} \mathcal{E}(t) &= N \frac{d\Phi_B(t)}{dt} = 2\pi r^2 \mu_0 n \frac{di(t)}{dt} \\ &= 2\pi r^2 \mu_0 n I_m \omega \cos(\omega t) \end{aligned}$$

$$\text{induced current} = \mathcal{E}(t)/R$$

$$\text{current amplitude} = 2\pi \mu_0 r^2 n I_m \omega / R$$

4. Failed candidates:

$$F = \frac{\mu_0}{2\pi} \frac{di^2}{L}$$

Equation shows increasing force if wires are moved apart (i.e. if d increases). No way!

$$F = \frac{\mu_0}{2\pi} \frac{Li^3}{d}$$

Dimensionally incorrect: newtons on left, newton-amperes on right. Never!

$$F = \frac{\mu_0}{2\pi} \frac{Li^2}{137d - L}$$

Force is infinite when $L = 137d$. Preposterous!

Correct result: The magnetic field due to the top long straight wire has magnitude

$$\frac{\mu_0}{2\pi} \frac{i}{r}$$

and direction given by the right-hand rule for \vec{B} from current. Thus the field at the bottom long straight wire is

$$\frac{\mu_0}{2\pi} \frac{i}{d}, \quad \text{directed into the page.}$$

The magnetic force $\vec{F} = i\vec{L} \times \vec{B}$ on the bottom wire thus has magnitude

$$\frac{\mu_0}{2\pi} \frac{Li^2}{d}$$

and, through the right-hand rule for cross products, direction downward (toward the bottom of the page).