Change in a cycle (*or* "There and back again")

(a) Isothermal change: $p_1V_1 = p_2V_2 = nRT_1$ so

$$\frac{p_2}{p_1} = \frac{V_1}{V_2} = \frac{1}{3.00} = 0.333$$

(b) Adiabatic change: $p_1V_1^\gamma = p_3V_3^\gamma$ so

$$\frac{p_3}{p_1} = \left(\frac{V_1}{V_3}\right)^\gamma = \frac{1}{(3.00)^{7/5}} = 0.215$$

(c) Adiabatic change: $T_1V_1^{\gamma-1} = T_3V_3^{\gamma-1}$ so

$$\frac{T_3}{T_1} = \left(\frac{V_1}{V_3}\right)^{\gamma-1} = \frac{1}{(3.00)^{2/5}} = 0.644$$

(d) For the isothermal expansion

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{nRT_1}{V}\,dV = nRT_1\int_{V_1}^{V_2}\frac{dV}{V} = nRT_1 \ln\left(\frac{V_2}{V_1}\right)$$

so

$$\frac{W}{nRT_1} = \ln\left(\frac{V_2}{V_1}\right) = \ln(3.00) = 1.10.$$

(e) For the isothermal expansion of an ideal gas there is no change in internal energy, so

$$\frac{Q}{nRT_1} = \frac{W}{nRT_1} = 1.10.$$

(f) For the isothermal expansion of an ideal gas there is no change in internal energy, $\Delta E_{\text{int}} = 0$.

(g) For the adiabatic compression $pV^\gamma = p_1V_1^\gamma$ so

$$\begin{aligned} W &= \int_{V_3}^{V_1} p\,dV = \int_{V_3}^{V_1} \frac{p_1V_1^\gamma}{V^\gamma}\,dV = p_1V_1^\gamma \int_{V_3}^{V_1}\frac{1}{V^\gamma}\,dV = p_1V_1^\gamma\left[-\frac{1}{(\gamma-1)V^{\gamma-1}}\right]_{V_3}^{V_1} \\ &= -\frac{p_1V_1^\gamma}{\gamma-1}\left[\frac{1}{V_1^{\gamma-1}} - \frac{1}{V_3^{\gamma-1}}\right] = -\frac{p_1V_1}{\gamma-1}\left[1-\left(\frac{V_1}{V_3}\right)^{\gamma-1}\right] \end{aligned}$$

and

$$\frac{W}{nRT_1} = -\frac{1}{\gamma-1}\left[1-\left(\frac{V_1}{V_3}\right)^{\gamma-1}\right] = -\frac{1}{2/5}\left[1-\frac{1}{(3.00)^{2/5}}\right] = -0.889.$$

(h) For the adiabatic compression $Q = 0$.

(i) For the adiabatic compression

$$\frac{\Delta E_{\text{int}}}{nRT_1} = \frac{Q}{nRT_1} - \frac{W}{nRT_1} = 0.889.$$

(j) For the cooling at constant volume to reduce pressure, $W = 0$.

(k) For the cycle $\Delta E_{\text{int}} = 0$, i.e. $\Delta E_{\text{int},1\to 2} + \Delta E_{\text{int},2\to 3} + \Delta E_{\text{int},3\to 1} = 0$, but we know $\Delta E_{\text{int},1\to 2}$ from part (f) and $\Delta E_{\text{int},3\to 1}$ from part (j). For the cooling at constant volume to reduce pressure,

$$\frac{\Delta E_{\text{int}}}{nRT_1} = -0.889.$$

(ℓ) Using $\Delta E_{\text{int}} = Q - W$ for this cooling,

$$\frac{Q}{nRT_1} = -0.889.$$

Grading: one point per lettered item for a total of 12 points.