Physics 103 Elementary Physics I

Model Solutions to First Examination, Fall 2023

1. *Runaway truck.* An out-of-control truck enters a 118 m long runaway truck ramp at speed 26.6 m/s. What is the minimum constant acceleration the truck must experience to stop on the ramp?

Solution: We are given speeds and distances, not times, so the most relevant equation is

$$\begin{aligned} v^2(x) &= v_0^2 + 2a_0(x - x_0) \\ 0 &= v_0^2 + 2a_0(\text{length}) \\ a_0 &= -\frac{v_0^2}{2(\text{length})} = -\frac{(26.6\text{ m/s})^2}{2 \times 118\text{ m}} = -3.00\text{ m/s}^2 \end{aligned}$$

⟦*Grading:* 2 points for $v(x)$ equation; 2 points for a_0 solution; 2 points for numerical solution; 2 points for three significant figures; 2 points for dimensions of numerical solution. Negative sign optional.⟧

2. *Lost in space.* A pebble requires 0.87 s to reach the ground after being dropped from rest at height of 1.8 m. What is the acceleration due to gravity? Which planet are you on?

Solution: We are given distances and times, not speeds, so the most relevant equation is (where t_S means "time when the pebble strikes the ground")

$$\begin{aligned} y(t) &= y_0 + v_0 t - \tfrac{1}{2} a_g t^2 \\ 0 &= y_0 - \tfrac{1}{2} a_g t_S^2 \\ a_g &= \frac{2y_0}{t_S^2} = \frac{2(1.8\text{ m})}{(0.87\text{ s})^2} = 4.8\text{ m/s}^2 \end{aligned}$$

Comparison to the information table (rounding to $g = 10$ m/s^2) suggests we are on planet TRAPPIST-1d.
⟦*Grading:* 2 points for $y(t)$ equation; 2 points for a_g solution; 2 points for numerical solution; 1 point for two significant figures; 1 points for dimensions of numerical solution; 2 points for comparison to table.⟧

4. *Cliff drop.* A pebble at rest drops from the top of a cliff. The time required to drop the first half of the cliff's height (t_h) is of course less than the time required to drop the entire height of the cliff (t_e), but how much less? Find the ratio t_h/t_e.

Solution: Call the cliff height H, the acceleration of gravity g. Set coordinates with origin at base of cliff, positive upward. Then the position is

$$\begin{aligned} x(t) &= x_0 + v_0 t + \tfrac{1}{2} a_0 t^2 \\ x(t) &= H - \tfrac{1}{2} g t^2. \end{aligned}$$

At half-way point

$$\tfrac{1}{2} H = H - \tfrac{1}{2} g t_h^2 \qquad \text{whence} \qquad t_h = \sqrt{H/g}.$$

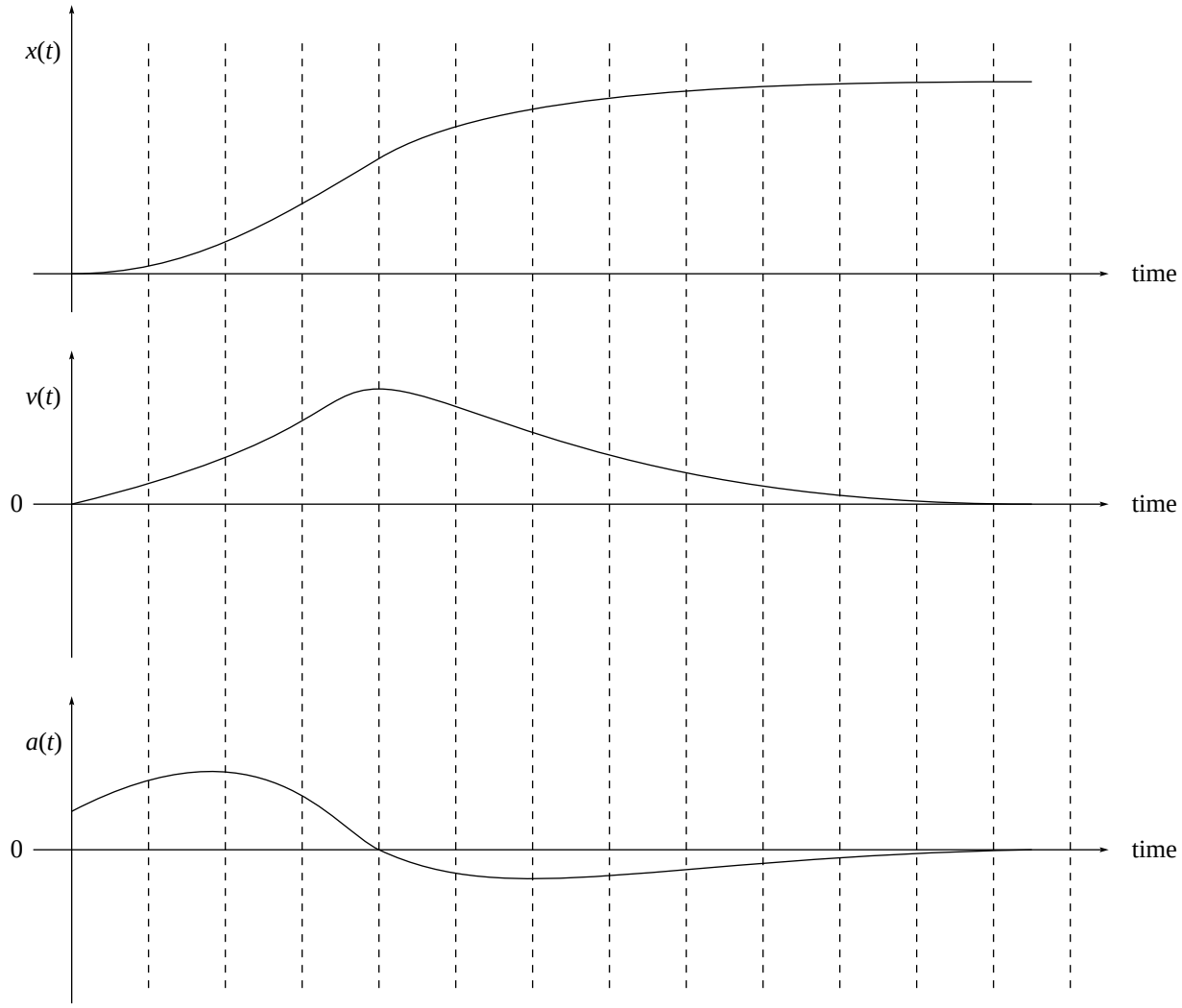
At entire drop

$$0 = H - \tfrac{1}{2} g t_e^2 \qquad \text{whence} \qquad t_e = \sqrt{2H/g}.$$

Thus $t_h/t_e = 1/\sqrt{2} \approx 0.707$. (It makes sense that the first half of the journey should take more than half the time, because the pebble travels slowly on the first half, faster on the second half.)

⟦*Grading:* 2 points for general $x(t)$; 2 points for $x(t)$ for this specific problem; 2 points for finding t_h; 2 points for finding t_e; 2 points for ratio.⟧

3. *Rocket-propelled sled.*



⟦*Grading:* 1 point each for these ten features: $v(t)$: always non-negative, starts at zero, ends at zero, maximum at or near the fourth vertical dashed line, goes up steeply, goes down gradually; $a(t)$: initial acceleration positive (not zero), positive to left of velocity maximum, zero at velocity maximum, negative to right of velocity maximum, larger magnitude on left.⟧