Oberlin College Physics 103, Fall 2023

Model Solutions to Assignment 2

Problems from *College Physics* by P.P. Urone and R. Hinrichs.

All these trajectory problems are solved based on these five equations:
x-motion (constant horizontal velocity):

$$v_x(t) = v_{x,0} \tag{1}$$
$$x(t) = x_0 + v_{x,0}t \tag{2}$$

y-motion (constant downward acceleration):

$$v_y(t) = v_{y,0} - gt \tag{3}$$
$$y(t) = y_0 + v_{y,0}t - \tfrac{1}{2}gt^2 \tag{4}$$
$$v_y^2(y) = v_{y,0}^2 - 2g(y - y_0) \tag{5}$$

Chapter 3, problem 26
(a) Speed striking ground. For horizontal component use equation (1). For vertical component use equation (5). Ball strikes when $y = y_0$, so $v_y^2 = v_{y,0}^2$, so

$$\text{striking speed} = \sqrt{v_x^2 + v_y^2} = \sqrt{(16 \text{ m/s})^2 + (12 \text{ m/s})^2} = \sqrt{(20 \text{ m/s})^2} = 20 \text{ m/s}.$$

(b) Ball strikes ground at time t_S when $v_y(t_S) = -12$ m/s. So use equation (3): $-12 \text{ m/s} = 12 \text{ m/s} - gt_S$, whence $t_S = (24 \text{ m/s})/g = 2.4$ s.
(c) At maximum height $v_y = 0$, so use equation (5): $0 = v_{y,0}^2 - 2gh_{\max}$, whence $h_{\max} = v_{y,0}^2/(2g) = (12 \text{ m/s})^2/(2g) = 7.3$ m.
⟦*Grading:* 1 point for free. Each part [(a), (b), and (c)] is worth three points: 1 point for setup; 1 point for execution to get the right number; 1 point for getting two significant figures and proper units.⟧

Chapter 3, problem 27 In this problem $v_{y,0} = 0$.
(a) How long in air? Strikes ground at time t_S, that is $y(t_S) = 0$, so use equation (4)

$$\begin{aligned} 0 &= y_0 - \tfrac{1}{2}gt_S^2 \\ \tfrac{1}{2}gt_S^2 &= y_0 \\ t_S &= \sqrt{2y_0/g} = \sqrt{2(60.0 \text{ m})/(9.81 \text{ m/s}^2)} = 3.50 \text{ s}. \end{aligned}$$

(b) Launch speed? Travels 100.0 m in 3.50 s so

$$v_{0,x} = \frac{100.0 \text{ m}}{3.50 \text{ s}} = 28.6 \text{ m/s}.$$

(c) Vertical velocity at striking ground comes from equation (5):

$$\begin{aligned} v_y^2(y) &= -2g(y - y_0) \\ v_y^2 &= -2g(0 - y_0) \\ v_y &= -\sqrt{2gy_0} = -\sqrt{2(9.81 \text{ m/s}^2)(60.0 \text{ m})} = -34.3 \text{ m/s}. \end{aligned}$$

(d) Horizontal velocity at striking ground is 28.6 m/s.
⟦*Grading:* 1 point for part (d); 3 points for each of (a), (b), and (c). For these three parts, 1 point for setup, 1 point for execution, 1 point for correct units and significant figures.⟧

Chapter 3, problem 34
In this problem,

$$\begin{aligned} v_{0,x} &= (30 \text{ m/s}) \cos 60^\circ = 15 \text{ m/s}, \\ v_{0,y} &= (30 \text{ m/s}) \sin 60^\circ = 26 \text{ m/s}. \end{aligned}$$

(a) To find the height H at time $t = 4.0$ s, use equation (4):

$$H = 1.5 \text{ m} + (26 \text{ m/s})(4.0 \text{ s}) - \tfrac{1}{2}g(4.0 \text{ s})^2 = 27 \text{ m}.$$

(b) The maximum height comes when $v_y = 0$, so use equation (5):

$$\begin{aligned} v_y^2 &= v_{0,y}^2 - 2g(y - y_0) \\ 0 &= v_{0,y}^2 - 2g(h_{\text{max}} - y_0) \\ v_{0,y}^2/(2g) &= h_{\text{max}} - y_0 \\ h_{\text{max}} &= v_{0,y}^2/(2g) + y_0 = 36 \text{ m} \end{aligned}$$

(c) Horizontal component of velocity is a constant 15 m/s. Vertical component obeys equation (5):

$$\begin{aligned} v_y^2 &= v_{0,y}^2 - 2g(y - y_0) \\ &= (26 \text{ m/s})^2 - 2g(27 \text{ m} - 1.5 \text{ m}) = 180.6 \text{ m}^2/\text{s}^2. \end{aligned}$$

(Because this is an intermediate result, I'm keeping four significant figures, to be rounded down at the final stage.) So the speed at impact is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15 \text{ m/s})^2 + 180.6 \text{ m}^2/\text{s}^2} = 20 \text{ m/s}.$$

⟦*Grading:* 1 point for free. Each part [(a), (b), and (c)] is worth three points: 1 point for setup; 1 point for execution to get the right number; 1 point for getting two significant figures and proper units.⟧

Chapter 3, problem 43
According to U&H equation (3.71), the range of a projectile is $v_0^2 \sin(2\theta_0)/g$. The maximum range comes when $\sin(2\theta_0) = 1$, so that maximum range is v_0^2/g. For $v_0 = 30$ m/s, this maximum range is 92 m, which is a bit less than the width of a soccer field.
⟦*Grading:* 5 points for use of range equation; 5 points for number.⟧

Chapter 3, problem 44

I will use: t_B for time when basketball reaches basket, x_B for distance to the basket, and h for the height of the basket above its launch point (2 feet).

I will apply equation (2) to the instant when the ball enters the basket:

$$x_B = v_{0,x} t_B.$$

And I will apply equation (4) to that same instant:

$$h = v_{0,y} t_B - \tfrac{1}{2} g t_B^2.$$

We know h, x_B, and g. The equations involve the unknown t_B, but we're not asked for t_B. So solve the first equation for t_B and plug into the second:

$$\begin{aligned} h &= \frac{v_{0,y}}{v_{0,x}} x_B - \frac{1}{v_{0,x}^2} \frac{x_B^2 g}{2} \\ \frac{h}{x_B} &= \frac{\sin\theta_0}{\cos\theta_0} - \frac{1}{\cos^2\theta_0} \frac{x_B g}{2v_0^2}. \end{aligned}$$

I can't think of any analytic way of solving this equation, so I wrote a spreadsheed evaluating it for every angle from 0° to 89°. (I knew that if I tried evaluating it at 90° I'd be dividing by zero.) The left hand side is $h/x_B = (2\text{ ft})/(10\text{ ft}) = 0.2$. The constant on the right is $x_B g/2v_0^2 = 0.225$. I found that the right hand side equaled 0.2 at 26° and 80°.

⟦*Grading:* 4 points for using equations (2) and (4); 4 points for setting up the equation involving angle; 2 points for any numerical solution.⟧

Chapter 3, problem 48

When you follow the suggestions, you find

$$a = \frac{v_{0,y}}{v_{0,x}}, \qquad b = -\frac{g}{2v_{0,x}^2}.$$

⟦*Grading:* 7 points for any reasonable start; 3 points for actually finding these results for a and b.⟧