Oberlin College Physics 212, Fall 2021 semester

Model Solutions to Assignment 11

Would you buy a used eigenfunction from this man?

a. Eigenfunction $\eta_3(x)$ should have three bumps, not four (two nodes, not three) (B).

b. At the shallow end of the well, where the classical particle is slow, the wavy part should have large amplitude [and long wavelength] but this candidate has small amplitude [and long wavelength] (C). The two tails should be equal, because E_4 is exactly as forbidden to the right as it is to the left, but the left tail of the candidate is larger (E).

c. Just to the left of the left-hand vertical dashed line, in the classically-forbidden region, the candidate eigenfunction curves toward the axis where is should curve away (A). [On the other hand, the left-hand tail should have the sharper cutoff, because E_5 is more forbidden to the left than to the right, and it does.]

d. Because the energy is more forbidden on the left than on the right, the tail should be shorter on the left than on the right (E).

Grading criteria for "used eigenfunction"

			points
a.	candidate has error	B	2
	replacement should have:		
	number of nodes in wavy (classically allowed) region	2	2
	amplitude in wavy (classically allowed) region	equal throughout	2
	wavelength in wavy (classically allowed) region	equal throughout	2
	tails in monotonic (classically prohibited) region	equal on both sides	2
b.	candidate has errors	C, E	2
	replacement should have:		
	number of nodes in wavy (classically allowed) region	3	2
	amplitude in wavy (classically allowed) region	increasing to right	2
	wavelength in wavy (classically allowed) region	increasing to right	2
	tails in monotonic (classically prohibited) region	equal on both sides	2
c.	candidate has error	A	2
	replacement should have:		
	number of nodes in wavy (classically allowed) region	4	2
	amplitude in wavy (classically allowed) region	increasing to right	2
	wavelength in wavy (classically allowed) region	increasing to right	2
	tails in monotonic (classically prohibited) region	longer on right	2
d.	candidate has error	E	2
	replacement should have:		
	number of nodes in wavy (classically allowed) region	5	2
	amplitude in wavy (classically allowed) region	decreasing to right	2
	wavelength in wavy (classically allowed) region	decreasing to right	2
	tails in monotonic (classically prohibited) region	longer on right	2

A doomed attempt

To factorize into space × spin form, *all* of the terms in square brackets would have to be proportional. Let's see if we can even get the first two terms in square brackets to be proportional: Can we select a value of c so that

$$\Big[\alpha|\uparrow\downarrow\uparrow\rangle + \beta|\uparrow\downarrow\downarrow\rangle\Big] = c\,\Big[\alpha|\uparrow\uparrow\downarrow\rangle + \beta|\uparrow\downarrow\downarrow\rangle\Big] \quad ?$$

To make the $|\uparrow\downarrow\downarrow\rangle$ terms match up on both sides, we must select $c = 1$. But then we would have to have $|\uparrow\downarrow\uparrow\rangle = |\uparrow\uparrow\downarrow\rangle$, which is certainly *false*.

⟦*Grading:* Any sort of reasonable strategy earns 7 points. Pulling off that strategy earns 3 more points.⟧

Questions

⟦*Grading:* 10 points for any decent attempt; 5 points for "I can't think of anything."; 0 points for no answer at all.⟧