Oberlin College Physics 212, Fall 2021

Model Solutions to Assignment 1: Space and Time in Relativity

Problem 1.5. Interval

$$
\begin{aligned}
(ct')^2 - x'^2 &= \left[\frac{ct-(V/c)x}{\sqrt{1-(V/c)^2}}\right]^2 - \left[\frac{x-(V/c)(ct)}{\sqrt{1-(V/c)^2}}\right]^2 \\
&= \frac{[ct-(V/c)x]^2 - [x-(V/c)(ct)]^2}{1-(V/c)^2} \\
&= \frac{[(ct)^2 - 2(V/c)x(ct) + (V/c)^2x^2] - [x^2 - 2(V/c)x(ct) + (V/c)^2(ct)^2]}{1-(V/c)^2} \\
&= \frac{[(ct)^2 + (V/c)^2x^2] - [x^2 + (V/c)^2(ct)^2]}{1-(V/c)^2} \\
&= \frac{\{(ct)^2 - (V/c)^2(ct)^2\} - \{x^2 - (V/c)^2x^2\}}{1-(V/c)^2} \\
&= \frac{\{1-(V/c)^2\}(ct)^2 - \{1-(V/c)^2\}x^2}{1-(V/c)^2} \\
&= (ct)^2 - x^2
\end{aligned}
$$

⟦*Grading:* 3 points for setting up the first line; 3 more points for some correct algebra; 4 more points for correctly reaching the final result.⟧

Problem 1.9. Galactic journey

Model solution is in its own file.

Problem 1.10. Flushing out an error

At time 1:45 into the video, the Jeopardy "answer" is

$$
\begin{aligned}
x^1 &= \frac{x - Vt}{\sqrt{1 - x^2/c^2}} \\
y^1 &= y \\
t &= \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}}
\end{aligned}
$$

This is a silly set of equations. The first one isn't even dimensionally correct! (You may think that all the rules of common sense are thrown out in relativity. No! Equations still need to be dimensionally consistent.) The last one shows t equal to an expression involving t. Dumb.

Clearly, this is supposed to be the Lorentz transformation,

$$
\begin{aligned}
x' &= \frac{x - Vt}{\sqrt{1 - (V/c)^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - Vx/c^2}{\sqrt{1 - (V/c)^2}}
\end{aligned}
$$

⟦*Grading:* 3 points for finding the silly equations (students don't need to reference the exact time stamp in the video); 3 more points for pointing out dimensional error; 4 more points for writing down correct Lorentz tansformation.

Notice that you cannot just say "that equation is wrong", you need to explain *how you know* that it's wrong (namely, because it's dimensionally inconsistent). As the syllabus says, "In writing your solutions, do *not* just write down the final answer. Show your reasoning and your intermediate steps. Describe (in words) the thought that went into your work as well as describing (in equations) the mathematical manipulations involved."

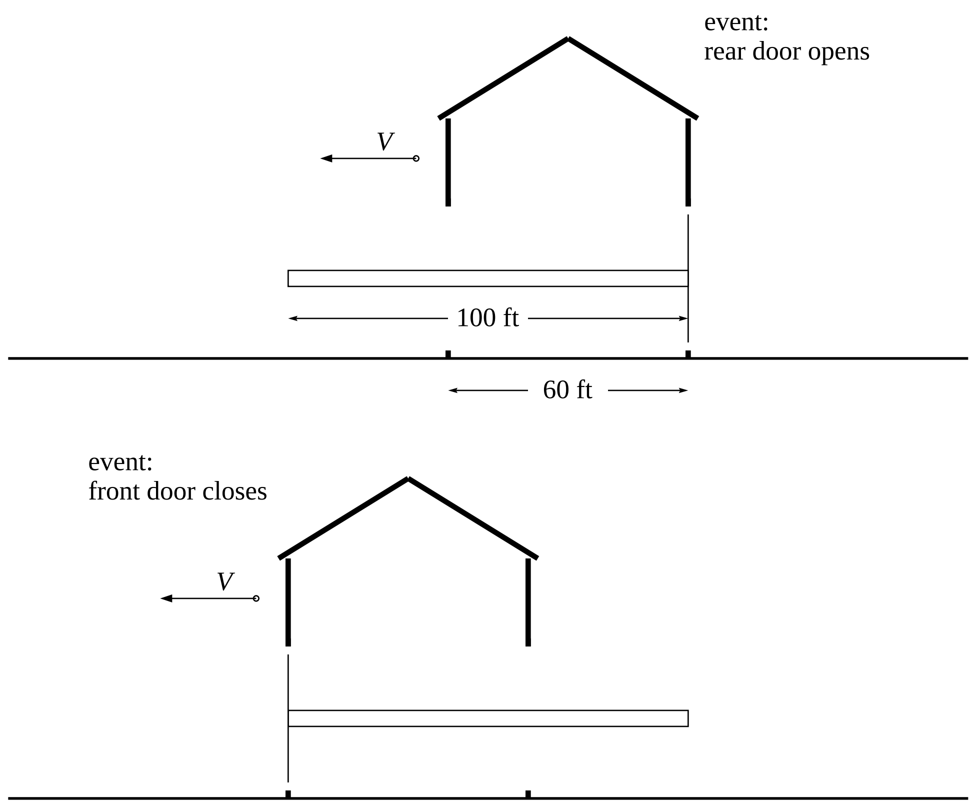
This is not nit-picking, it is a sensible rule for all rational discourse. Suppose, for example, that in an American Literature course you are assigned to write a paper on the influence of Herman Melville. After considerable reading and analysis, you conclude that *Moby Dick* changed the entire landscape of the American novel. If you write an essay consisting only of that single concluding sentence — "*Moby Dick* changed the entire landscape of the American novel." — you'll earn an F for the assignment, despite the fact that everything you write is true!⟧

Problem concerning "Pole in the barn"

In the vaulter's frame:

- The pole is stationary and the barn moves left.
- The pole is 100 feet long and the barn is 60 feet wide.
- The two events occur in the opposite sequence.
- Barn clocks tick slowly, but no clocks are shown so this doesn't affect the sketch.

Vaulter's frame:



⟦*Grading:* 1 point for each element in the verbal description; 6 points for the two figures; if the two figures are drawn superimposed into one figure, but otherwise correct, deduct 1 point.⟧