Oberlin College Physics 110, Fall 2011

Model Solutions to Assignment 11

1. A trip to the grocery store.

Jane Morowitz travels at speed $V = \frac{4}{5}c$ to a grocery store located 1200 feet from her home. As she starts her journey, her wristwatch, her home's wall clock, and the grocery store's wall clock all read noon. (Note that $\sqrt{1-(V/c)^2} = \frac{3}{5}$.)

a. How much time (in the earth's frame) does it take her to reach the grocery store?

Answer:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{1200 \text{ ft}}{(4/5)(1 \text{ ft/nan})} = 1500 \text{ nan}$$

b. How much time ticks off on her wristwatch during this journey?

Answer: Her clock is ticking slowly (time dilation), so it ticks off a smaller time

$$T_0 = T\sqrt{1-(V/c)^2} = (1500 \text{ nan})\tfrac{3}{5} = 900 \text{ nan}.$$

c. In Jane's frame, the grocery store moves towards her. How far must it travel to reach her?

Answer: It must travel the length-contracted distance of

$$L = L_0\sqrt{1-(V/c)^2} = (1200 \text{ ft})\tfrac{3}{5} = 720 \text{ ft}.$$

d. Using your result for part (c) and the fact that distance = speed × time, find out how much time it takes for the grocery store to reach Jane. Compare to your result from part (b).

Answer:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{720 \text{ ft}}{(4/5)(1 \text{ ft/nan})} = 900 \text{ nan}$$

The same as the result of part (b), as of course it must.

e. In Jane's frame, the grocery store's wall clock is moving, so it ticks slowly. How much time does it tick off during its journey to reach Jane?

Answer: In Jane's frame the wall clock is moving, and hence ticking slowly. It ticks off a smaller time

$$T_0 = T\sqrt{1-(V/c)^2} = (900 \text{ nan})\tfrac{3}{5} = 540 \text{ nan}.$$

f. When Jane reaches the grocery store, its wall clock will read the time you found in part (a). How can it read this time when it has ticked off only the amount of time you determined in part (e)?

Answer: In Jane's frame the two wall clocks aren't synchronized. At the start of her journey her home clock reads noon, but the store's clock — the rear clock — is set ahead by

$$\frac{L_0 V}{c^2} = \frac{(1200 \text{ ft})(\frac{4}{5}c)}{(1 \text{ ft/nan})c} = 960 \text{ nan}.$$

Since this clock starts off reading 960 nans, and ticks off 540 nans, at the end of the journey it reads 1500 nans.

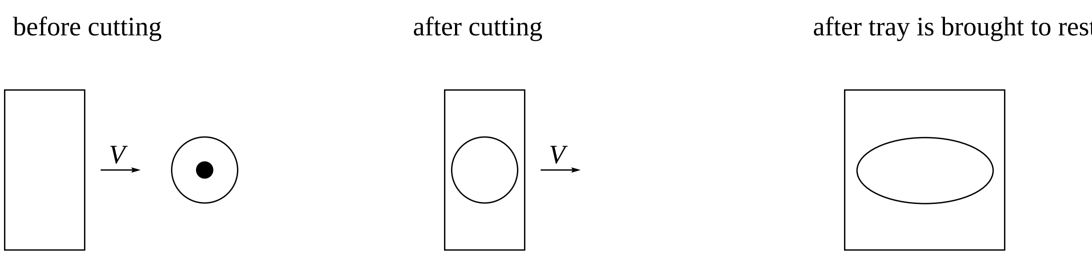
3. Engine trouble.

If there were a clock at the driver's seat and a clock at the tail pipe (and those two clocks were synchronized in the car's frame) then the two clocks would strike noon simultaneously in the car's frame. In the earth's frame, the rear clock would be set ahead, i.e. the rear clock would strike noon first. Just as the tail pipe-clock strikes noon before the driver's-seat clock strikes noon, so the puff of smoke emerges before the driver coughs.

Or, in a single sentence: "The rear event happens before the front event."

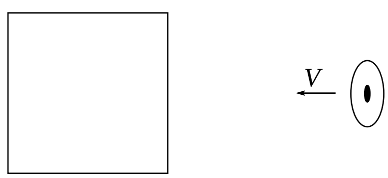
4. Relativistic baking.

Analysis in the baker's frame: The cookie cutter is circular, the tray of dough is rectangular (length contracted in the direction of motion).



When the tray is brought to rest, the hole (and cookie) will be an oval longer in the direction of motion.

Preliminary analysis in the tray's frame: The cookie cutter is an oval (length contracted in the direction of motion), the tray of dough is square.

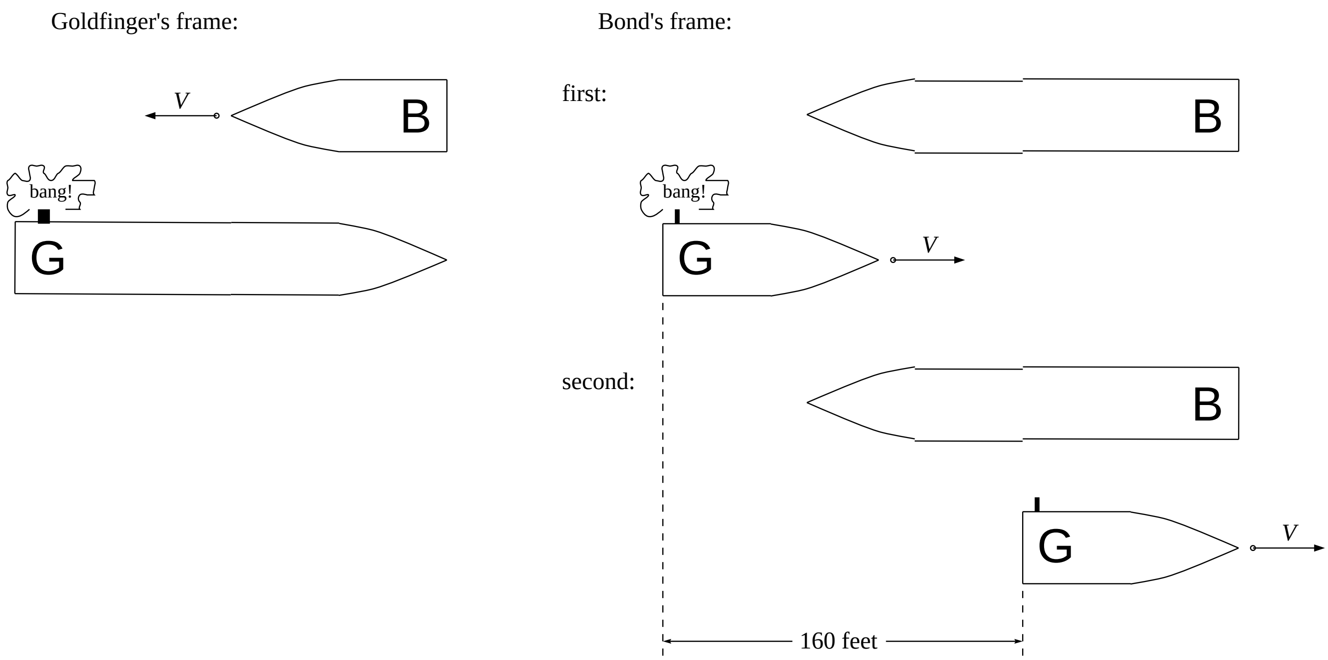


The cookie stamped out will be the same shape as the cookie cutter, so it will be shorter in the direction of motion.

No! These two results contradict each other. . . they can't both be right!

Corrected analysis in the tray's frame: Is it really true that "the cookie stamped out will be the same shape as the cookie cutter"? No. This will be true only if every piece of the cookie cutter hits the dough simultaneously. In the baker's frame this is true, so in the tray's frame it can't be true! In the tray's frame, first the rear (right) edge hits the dough and raises up, then a little while later the front (left) edge hits the dough and raises up. Since the cookie cutter travels left between these two events, the resulting cookie will be elongated in the direction of motion.

5. Spy versus spy.



Note that $\sqrt{1-(V/c)^2} = \frac{3}{5}$.

a. In Goldfinger's frame, Bond's ship is length contracted to $(150 \text{ ft})\frac{3}{5} = 90$ ft.

b. So the bullet flies 60 feet (150 ft − 90 ft) in front of Bond's bow.

c. In Bond's frame, the rear event is the gun firing, so first the gun will fire, and later the nose and tail will line up.

d. In Bond's frame, Goldfinger's tail clock is ahead by

$$\frac{L_0 V}{c^2} = (150)\tfrac{4}{5} \text{ nan} = 120 \text{ nan}.$$

e. In Bond's frame, Goldfinger's clocks tick slowly. So when they tick off 120 nans, a longer time of $(120 \text{ nan})\frac{5}{3} = 200$ nan elapses.

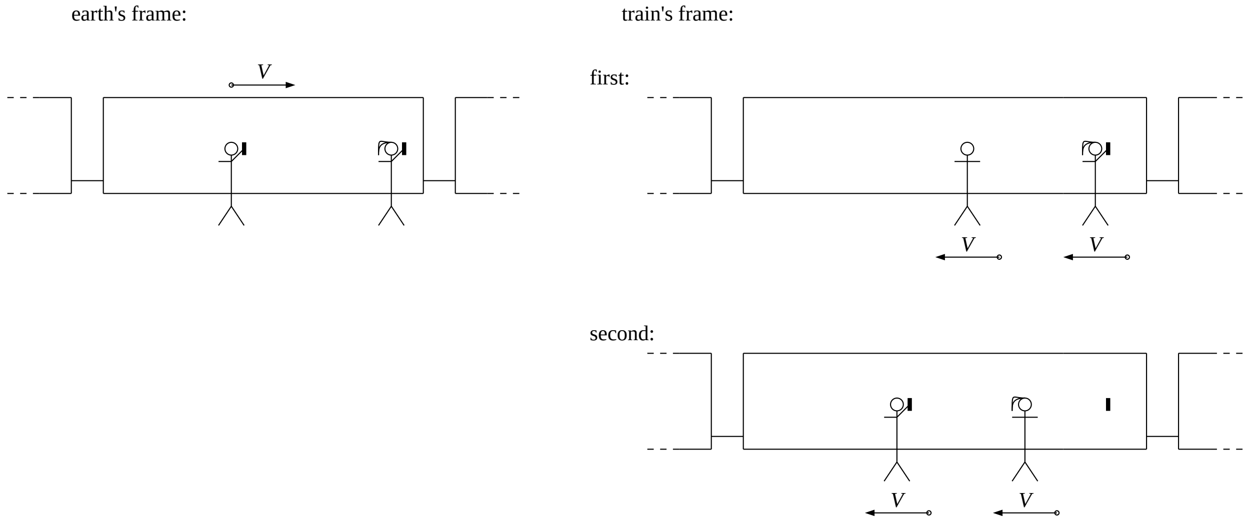
f. During this time interval, Goldfinger's ship moves $(\frac{4}{5}c)(200 \text{ nan}) = 160$ ft.

g. From the figure above right, the distance between Goldfinger's bullet and Bond's bow is

$$160 \text{ ft} + 90 \text{ ft} - 150 \text{ ft} = 100 \text{ ft}.$$

h. $L_0 = 100$ ft and $L = 60$ ft, so $L = L_0\sqrt{1-(V/c)^2}$.

6. American graffiti.



In the reference frame of the train:

a. The rear event, Jennifer's marking, happens first.

b. When Jennifer's watch reads noon, Paul's reads

$$\frac{L_0 V}{c^2} = \frac{(100\text{ ft})(\frac{3}{5}c)}{c^2} = 60\text{ nan}$$

before noon. But Paul's moving clock ticks slowly, so a longer time

$$(60\text{ nan})\tfrac{5}{4} = 75\text{ nan}$$

elapses before Paul marks. The train has only one mark for 75 nans.

c. During this time Paul travels a distance of

$$\text{speed} \times \text{time} = (\tfrac{3}{5}c)(75\text{ nan}) = 45\text{ ft}.$$

d. The two miscreants are separated by a length-contracted distance of 80 feet, so the two marks on the train are separated by

$$45\text{ ft} + 80\text{ ft} = 125\text{ ft} = (100\text{ ft})/\sqrt{1-(V/c)^2}\,.$$

10. Two fast women.

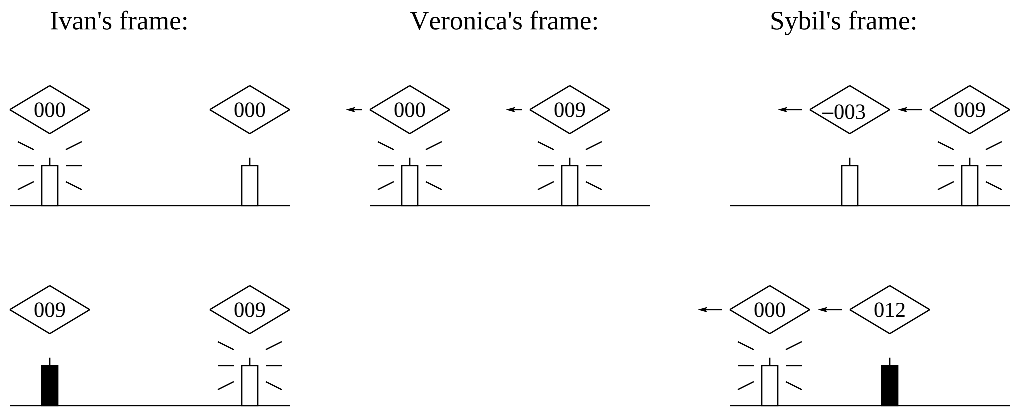
Solution using time dilation, length contraction, and the relativity of simultaneity: Suppose there's a clock located at each firecracker: clocks and firecrackers are all fixed in Ivan's frame. The left firecracker explodes when its clock reads 000, the right firecracker explodes when its clock reads 009.

In Ivan's frame these two clocks are synchronized. The right explosion happens 9 nan after the left explosion.

In Veronica's frame the rear clock — the right hand clock — is set ahead of the front clock by $L_0 V/c^2 = (15 \text{ ft})(\frac{3}{5}c)/c^2 = 9$ nan. So in Veronica's frame the two explosions are simultaneous.

In Sybil's frame the rear clock is set ahead of the front clock by $L_0 V/c^2 = (15 \text{ ft})(\frac{4}{5}c)/c^2 = 12$ nan. Thus the *left* explosion happens first in Sybil's frame. (See sketch below.) The clocks tick off 3 nan between these two explosions, so you might think that 3 nan have elapsed… but no, those clocks are moving clocks and they tick slowly. The time elapsed in Sybil's frame is the larger time

$$T = \frac{T_0}{\sqrt{1-(V/c)^2}} = \frac{3 \text{ nan}}{3/5} = 5 \text{ nan}.$$



Solution using the Lorentz transformation: The two explosions in Ivan's frame are separated by a distance $\Delta x = 15$ feet and by a time $\Delta t = 9$ nans. What is the time interval between these two events in Veronica's frame? In Sybil's frame? *Answer:* Use

$$\Delta t' = \frac{\Delta t - V\Delta x/c^2}{\sqrt{1-(V/c)^2}}$$

with $V = \frac{3}{5}c$ to find $\Delta t' = 0$ nan for Veronica. Similarly with $V = \frac{4}{5}c$ to find $\Delta t' = -5$ nan for Sybil.