Oberlin College Physics 110, Fall 2011

Model Solutions to Assignment 8

HRW problem 8–12: *Snowball.*

Energy at top of cliff is $mgh + \frac{1}{2}mv_i^2$.
Energy at bottom of cliff is $\frac{1}{2}mv_f^2$.
The forces are conservative, so these two energies are equal and $v_f^2 = v_i^2 + 2gh$.

a. Plugging in values gives a final speed of 21.0 m/s.
b and **c.** It is clear from the expression that the final speed is independent of both mass and launch angle.

HRW problem 8–22: *Ski jump.*
Stage I: Find speed of skier as he leaves the ramp — use energy conservation.
Let m represent the mass of the skier, H the height of the mountain.

Energy at top of the mountain is mgH.
Energy as he leaves the ramp is $\frac{1}{2}mv^2$.
The normal force does no work, and friction is negligible, so these two energies are equal and $v^2 = 2gH$.

Thus the speed of the skier leaving the ramp is

$$v_{\text{ramp}} = \sqrt{2gH}.$$

Stage II: Find height of the jump — use kinematics.
Let θ represent the angle of the ski jump. (In our case, $\theta = 28°$.)
Recall that in trajectories the horizontal and vertical motions are independent. In this problem we only care about the vertical motion. Also, in this problem we want to relate velocities and distances — we're not concerned about time. So the equation to use is clearly

$$v_y^2 = v_{y,0}^2 - 2gy.$$

The vertical component of the initial velocity is

$$v_{y,0} = v_{\text{ramp}} \sin\theta,$$

and the maximum height $y = h$ is achieved when $v_y = 0$. Thus

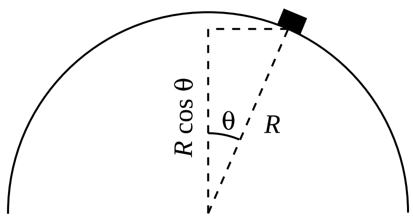
$$h = \frac{v_{y,0}^2}{2g} = \frac{(v_{\text{ramp}} \sin\theta)^2}{2g} = H \sin^2\theta.$$

(Notice that this gives the correct result when $\theta = 0°$ and when $\theta = 90°$.)

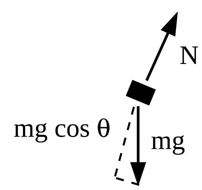
Answers: Plugging in the numbers from the problem, $h = 4.4$ m. This height is independent of the mass of the skier *and* of the value of g.

HRW problem 8–34: *Ice mound.*

geometry diagram:



free body diagram:



Needed: the normal force N on the boy, which vanishes when the boy leaves the ice.

First use force techniques. Apply $\sum \vec{F} = m\vec{a}$ in the radial direction:

$$mg\cos\theta - N = ma = m\frac{v^2}{R}$$

so

$$N = mg\cos\theta - \frac{mv^2}{R}. \tag{1}$$

But what is mv^2?

Find this using energy techniques.

$$\begin{aligned} E &= \tfrac{1}{2}mv^2 + mgR\cos\theta \\ &= E_{\text{initial}} = mgR \end{aligned}$$

Thus

$$mv^2 = 2mgR(1-\cos\theta). \tag{2}$$

Combining equations (1) and (2) gives

$$\begin{aligned} N &= mg\cos\theta - 2mg(1-\cos\theta) \\ &= mg(3\cos\theta - 2). \end{aligned}$$

The boy leaves the ice when $N = 0$, that is when $\cos\theta = \frac{2}{3}$. Since the height is $R\cos\theta$, the boy leaves the ice at a height of $\frac{2}{3}R$.

HRW problem 8–36: *Spring gun.*

Stage I: The spring is compressed by distance d, then expands to its relaxed length, giving the marble a velocity v_0. What is v_0? This is an energy conservation problem:

$$\begin{aligned} E_{\text{initial}} &= \tfrac{1}{2}kd^2 + \tfrac{1}{2}m(0)^2 \\ E_{\text{final}} &= \tfrac{1}{2}k(0)^2 + \tfrac{1}{2}mv_0^2 \end{aligned}$$

So

$$v_0 = \sqrt{k/m}\, d \tag{1}$$

Stage II: The marble leaves the spring with horizontal velocity v_0 from a height H above the floor. How far to the right of the launch point does it land? This is a trajectory problem:

$$\begin{aligned} x(t) &= v_0 t \\ y(t) &= H - \tfrac{1}{2}gt^2 \end{aligned}$$

The ball hits the floor when $y(t) = 0$, that is when $t = \sqrt{2H/g}$, at which time $x(t)$ is

$$x_{\text{hit}} = v_0\sqrt{2H/g}. \tag{2}$$

Combining equations (1) and (2) gives

$$x_{\text{hit}} = \sqrt{\frac{2kH}{mg}}\, d.$$

(This equation makes sense: correct dimensions, increasing x_{hit} with increasing H, etc.) We don't know k, H, or m, but we do know that there's a (dimensionless) constant C such that

$$x_{\text{hit}} = Cd.$$

Bobby's experiment discovers that if $d = 1.10$ cm, then $x_{\text{hit}} = 193$ cm. From this we conclude that $C = 175$. Thus to make $x_{\text{hit}} = 220$ cm, Rhoda must set a d of 1.25 cm.

HRW problem 8–40: *Oxygen molecule.*

The potential energy is

$$U(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$$

so the force is

$$F(r) = -\frac{dU(r)}{dr} = -\left(\frac{-12A}{r^{13}} - \frac{-6B}{r^7}\right) = \frac{12A}{r^{13}} - \frac{6B}{r^7}.$$

(a) This force is zero at the distance r_{eq} such that $F(r_{eq}) = 0$. This happens when

$$\frac{2A}{r_{eq}^{13}} = \frac{B}{r_{eq}^7}$$

or

$$r_{eq} = (2A/B)^{1/6}.$$

(b) For small values of r, r^7 is small, but r^{13} is *really* small. Thus $1/r^7$ is big, but $1/r^{13}$ is *really* big. Consequently,

$$\text{for } r \ll r_{eq}, \quad \frac{12A}{r^{13}} \gg \frac{6B}{r^7}$$

whence the force is positive, i.e. points towards larger values of r, i.e. is repulsive.

(c) For large values of r, the reasoning is parallel: r^7 is large, but r^{13} is *really* large. Thus $1/r^7$ is small, but $1/r^{13}$ is *really* small. Consequently,

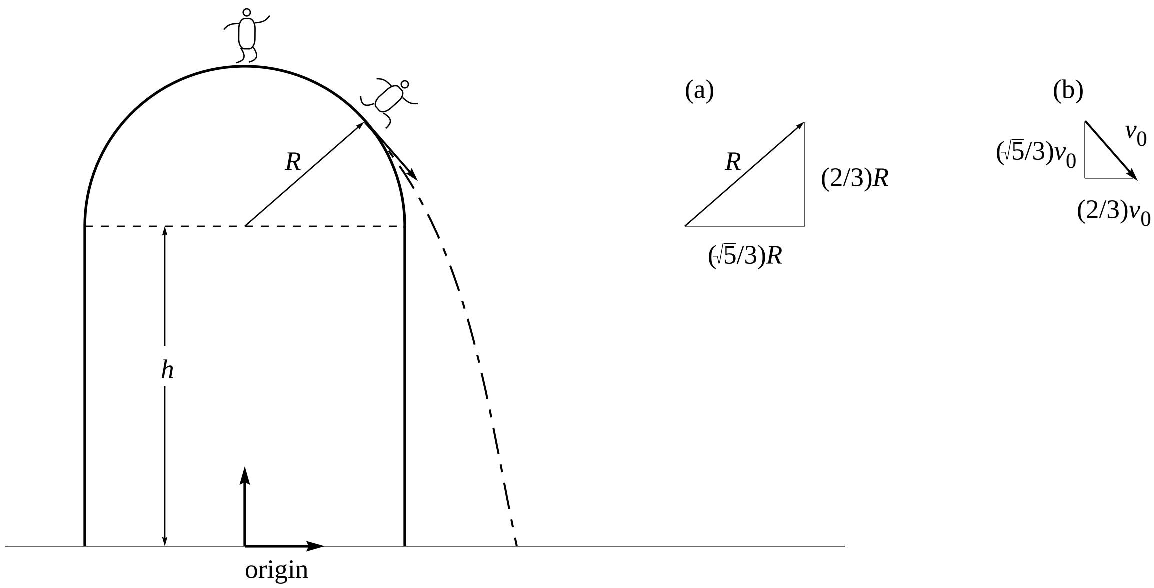
$$\text{for } r \gg r_{eq}, \quad \frac{12A}{r^{13}} \ll \frac{6B}{r^7}$$

whence the force is negative, i.e. points towards smaller values of r, i.e. is attractive.

Additional problem 81: *Daredevil astronomer.*

This is a pretty formidable problem and if you just jump into it without a strategy you'll almost certainly get lost. But if you remember to look before you leap, to use symbols rather than numbers, and generally to use all the problem solving techniques we've discussed this semester, then you'll find the problem to be challenging but not overwhelming, and a lot of fun as well.

Look before you leap: This problem breaks into two stages: the motion while the astronomer skates on the dome, and the motion after he or she leaves the dome. The first stage is exactly the problem we solved in "Ice mound". The second stage is a trajectory problem and we know how to solve any such problem given the initial position and velocity (i.e. the position and velocity at which the astronomer leaves the dome).



Stage I: Rolling down the dome. The astronomer leaves the dome 1/3 of the way down. The triangle at subfigure (a) shows that this is $(2/3)R$ above the base of the dome, and $(\sqrt{5}/3)R$ to the right of the center of the dome. (Pythagorean theorem.) That is, he begins free flight at

$$x_0 = (\sqrt{5}/3)R, \qquad y_0 = h + (2/3)R.$$

What is his velocity when he leaves? The magnitude comes from energy conservation:

at top of dome energy is $mg(h + R)$
upon leaving dome energy is $\frac{1}{2}mv_0^2 + mg(h + \frac{2}{3}R)$
so $\frac{1}{2}mv_0^2 = mg\frac{1}{3}R$
and $v_0 = \sqrt{\frac{2}{3}gR}$.

The velocity components come from the geometry of subfigure (b). (The triangle in (b) is similar to the triangle in (a).) We have

$$v_{0,x} = \frac{2}{3}\sqrt{\frac{2}{3}gR}, \qquad v_{0,y} = -\frac{\sqrt{5}}{3}\sqrt{\frac{2}{3}gR} = -\frac{1}{3}\sqrt{\frac{10}{3}gR}.$$

Stage II: Free flight. A particle (or an astronomer) is launched at (x_0, y_0) with velocity $(v_{0,x}, v_{0,y})$. Where does it (or he or she) hit the ground?

$$\begin{aligned} x(t) &= x_0 + v_{0,x}t \\ y(t) &= y_0 + v_{0,y}t - \tfrac{1}{2}gt^2 \end{aligned}$$

It hits at time t_{hit} when $y(t_{\text{hit}}) = 0$.

$$\begin{aligned} 0 &= y_0 + v_{0,y} t_{\text{hit}} - \tfrac{1}{2} g t_{\text{hit}}^2 \\ t_{\text{hit}} &= \frac{-v_{0,y} \pm \sqrt{v_{0,y}^2 - 4(-g/2) y_0}}{2(-g/2)} \\ &= \frac{v_{0,y} \mp \sqrt{v_{0,y}^2 + 2 g y_0}}{g} \end{aligned}$$

To get a positive t_{hit}, choose the + sign. Plug this result into the equation for $x(t)$ to find

$$x(t_{\text{hit}}) = x_0 + \frac{v_{0,x}}{g} \left(v_{0,y} + \sqrt{v_{0,y}^2 + 2 g y_0} \right).$$

Put together. Plug the values from the end of stage I into the general result for x_{hit}:

$$\begin{aligned} x_{\text{hit}} &= \tfrac{\sqrt{5}}{3} R + \tfrac{2}{3} \sqrt{\tfrac{2}{3} g R} \left(\frac{1}{g} \right) \left(-\tfrac{1}{3} \sqrt{\tfrac{10}{3} g R} + \sqrt{\tfrac{1}{9} \tfrac{10}{3} g R + 2 g h + 2 g \tfrac{2}{3} R} \right) \\ &= \tfrac{\sqrt{5}}{3} R + \tfrac{2}{3} \sqrt{\tfrac{2}{3} R} \left(-\tfrac{1}{3} \sqrt{\tfrac{10}{3} R} + \sqrt{\tfrac{46}{27} R + 2h} \right) \\ &= \left[\tfrac{\sqrt{5}}{3} + \tfrac{2}{3} \sqrt{\tfrac{2}{3}} \left(-\tfrac{1}{3} \sqrt{\tfrac{10}{3}} + \sqrt{\tfrac{46}{27} + 2(h/R)} \right) \right] R \\ &= \frac{1}{3} \left[\sqrt{5} - \frac{2}{9} \sqrt{20} + \frac{2}{9} \sqrt{92 + 108(h/R)} \right] R. \end{aligned}$$

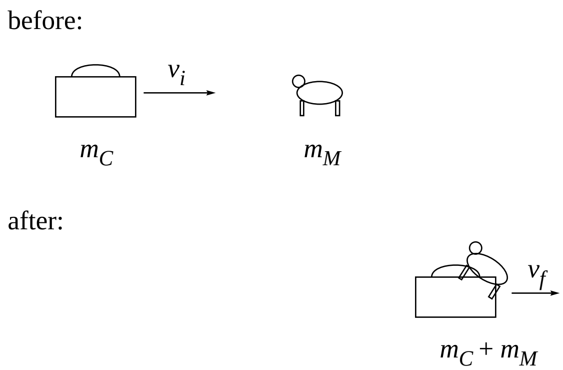
Note that the hit position is independent of g, just as it was in the "pendulum challenge" lab. In our case $(h/R) = 2$ so

$$x_{\text{hit}} = \frac{1}{3} \left[\sqrt{5} - \frac{2}{9} \sqrt{20} + \frac{2}{9} \sqrt{308} \right] R = 1.71\, R.$$

But we want the distance from the observatory wall, which is

$$x_{\text{hit}} - R = 0.71\, R = 5.7 \text{ meters}.$$

HRW problem 9-53: *Car hits moose.*



This is an inelastic collision.

$$\text{initial KE:} \quad \tfrac{1}{2} m_C v_i^2 = \frac{p^2}{2m_C} \qquad \text{final KE:} \quad \tfrac{1}{2}(m_C + m_M) v_f^2 = \frac{p^2}{2(m_C + m_M)}$$

Note there's no need to distinguish initial momentum from final momentum, because momentum is conserved.

$$\text{KE lost:} \quad \frac{p^2}{2m_C} - \frac{p^2}{2(m_C + m_M)}$$

So the fraction of KE lost is

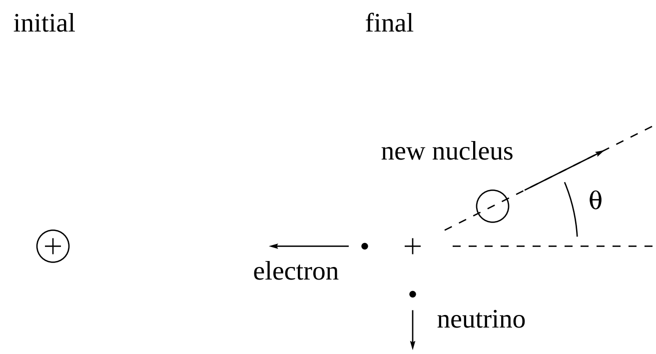
$$\frac{\dfrac{p^2}{2m_C} - \dfrac{p^2}{2(m_C + m_M)}}{\dfrac{p^2}{2m_C}} = \frac{\dfrac{1}{m_C} - \dfrac{1}{m_C + m_M}}{\dfrac{1}{m_C}} = \frac{m_C + m_M - m_C}{m_C + m_M} = \frac{m_M}{m_C + m_M} = \frac{1}{m_C/m_M + 1}$$

a. For moose, fraction of KE lost is $1/(\frac{10}{5} + 1) = 33\%$.

b. For camel, fraction of KE lost is $1/(\frac{10}{3} + 1) = 23\%$.

c. In general, decreasing m_M will decrease the fraction of KE lost.

HRW problem 9–90: *Neutrino* (omit part c).



The three momentum arrows shown on the right must sum to zero.
Thus the magnitude of the momentum of the recoiling new nucleus is

$$\sqrt{(12)^2 + (6.4)^2} \times 10^{-23} \text{ kg·m/s} = 14 \times 10^{-23} \text{ kg·m/s}.$$

The angle θ shown in the figure has

$$\tan(\theta) = \frac{6.4}{12} \quad \text{whence} \quad \theta = 28°.$$

Angle between new nucleus and neutrino is 118°; angle between new nucleus and electron is 152°.