Oberlin College Physics 110, Fall 2011

Model Solutions to Assignment 7

HRW problem 7–13: *Luge.*

I'm going to solve this problem in a different sequence from the one suggested in the book, because I find this sequence easier.

a. $F = (85 \text{ kg})(-2.0 \text{ m/s}^2) = -1.7 \times 10^2$ N.

c. work $= \Delta(\text{K.E.}) = (\text{K.E.})_{\text{final}} - (\text{K.E.})_{\text{initial}} = 0 - \frac{1}{2}mv_0^2 = -58 \times 10^3$ J

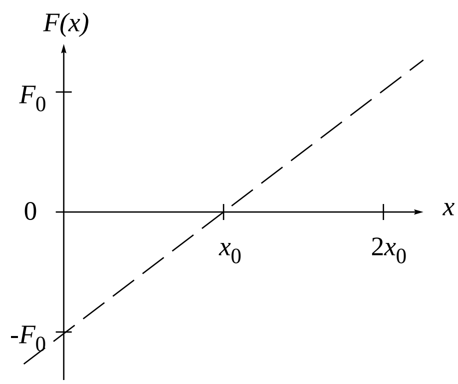
b. For a uniform force, work = force × distance, so

$$\text{distance} = \frac{\text{work}}{\text{force}} = 340 \text{ m}.$$

d, **e**, and **f.** If the deceleration doubles, then the force doubles, the work remains the same, and the distance halves.

HRW problem 7–35: *Work done by a variable force.*

Principle: Work done is $\int \vec{F} \cdot d\vec{x}$.



$$\begin{aligned}
\text{work} &= \int_0^{2x_0} F_0 \left(\frac{x}{x_0} - 1 \right) dx \\
&= F_0 \left[\frac{1}{2} \frac{x^2}{x_0} - x \right]_0^{2x_0} \\
&= F_0 \left[\frac{1}{2} \frac{4x_0^2}{x_0} - 2x_0 \right] \\
&= 0
\end{aligned}$$

HRW problem 7–62: *Shock absorber.*

Note: Take positive direction upward, origin at top of the relaxed spring.

Call the compression of spring $L = 0.12$ m.

a. Work done by gravity is

$$\int \vec{F}_{\text{grav}} \cdot d\vec{x} = \int (-mg\,\hat{\jmath}) \cdot (\hat{\jmath}\,dx) = \int_0^{-L} (-mg)\,dx = (-mg)[x]_0^{-L} = mgL.$$

(As always, gravity does positive work on an object moving down, negative work on an object moving up.)

b. Work done by the spring is

$$\int \vec{F}_{\text{spring}} \cdot d\vec{x} = \int (-kx\,\hat{\jmath}) \cdot (\hat{\jmath}\,dx) = \int_0^{-L} (-kx)\,dx = (-k)\left[\tfrac{1}{2}x^2\right]_0^{-L} = -\tfrac{1}{2}kL^2.$$

(Because the spring force points opposite to the motion, the work done by the spring is negative.)

c. The total work done is the change in kinetic energy,

$$mgL - \tfrac{1}{2}kL^2 = \Delta(\text{K.E.}) = (\text{K.E.})_{\text{final}} - (\text{K.E.})_{\text{initial}} = 0 - \tfrac{1}{2}mv_0^2,$$

so the initial velocity is given through

$$v_0^2 = \frac{k}{m}L^2 - 2gL.$$

Plugging in numbers gives $v_0 = 3.5$ m/s.

d. Rewrite the above equation as

$$\frac{k}{m}L^2 - 2gL - v_0^2 = 0$$

and solve using the quadratic formula, $(-b \pm \sqrt{b^2 - 4ac})/2a$, to find

$$L = \frac{2g \pm \sqrt{4g^2 + 4(k/m)v_0^2}}{2(k/m)} = \frac{1 \pm \sqrt{1 + (k/mg)(v_0^2/g)}}{(k/mg)}.$$

Take the positive sign so that the compression will be positive. Using this formula, an initial velocity of $v_0 = 7.0$ m/s gives $L = 0.23$ m.

HRW problem 7–54: *Work done by a variable force.*

Principle: Work done (area under curve) is change in kinetic energy.

Qualitative picture: The body starts out moving to the right and the force starts out pushing to the right. However once the body moves beyond the $x = 1$ m mark, the force is to the left. The body will slow down, stop, and eventually move to the left.

Initial kinetic energy $= \frac{1}{2}mv^2 = 16$ J. From the "area under curve" idea:

Work done in moving from $x = 0$ m to $x = 1$ m is 2 J.

Work done in moving from $x = 1$ m to $x = 2$ m is -2 J.

Work done in moving from $x = 2$ m to $x = 3$ m is -4 J.

Work done in moving from $x = 3$ m to $x = 4$ m is -4 J.

So the K.E. at 3 m is 12 J, the K.E. at 4 m is 8 J. The maximum K.E., found at $x = 1$ m, is 18 J.

HRW problem 7–65: *Pulley.*

a. As with the "window washer" problem, the tension is half the weight: $F = \frac{1}{2}mg = 98$ N.

b. To lift the canister by 2.0 cm, you must take 4.0 cm of cord out of play. (That's 2.0 cm from between the ceiling and the lower pulley, plus 2.0 cm from between the lower pulley and the upper pulley.)

c. The work done by your hand is

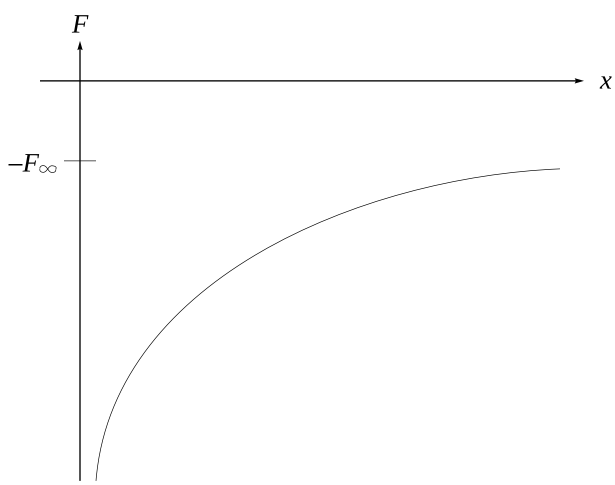
$$F \times \text{ (distance moved by hand) } = (98\text{ N}) \times (0.040\text{ m}) = 3.9\text{ J}.$$

d. The work done by gravity is

$$-mg \times \text{ (distance moved by canister) } = -(2 \times 98\text{ N}) \times (0.020\text{ m}) = -3.9\text{ J}.$$

Moral of the story: Net work done is zero, and there's no change in kinetic energy.

Quark-quark interactions.
a. For large separations, the QCD force approaches $-F_\infty$, the force becomes more and more negative as the separation becomes smaller, and the force approaches $-\infty$ as the separation vanishes.

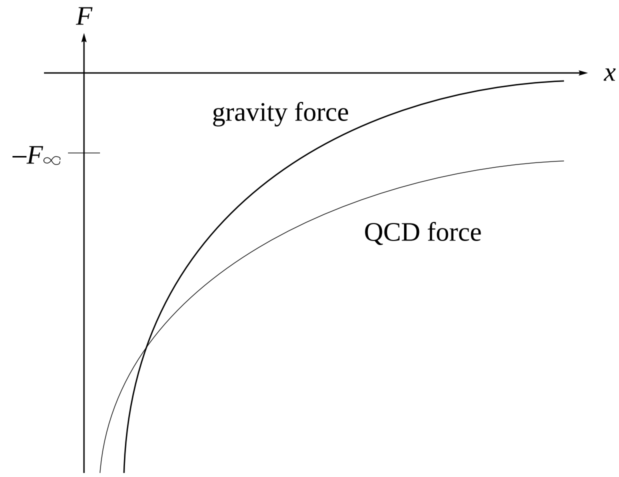


In contrast, the gravitational force vanishes for large separations. For small separations x, the force again goes to $-\infty$, but which force goes to $-\infty$ faster? When x is very small,

$$\frac{1}{x^2} \gg \frac{1}{x},$$

so for sufficiently small values of x

$$|F_G| \gg |F_Q|.$$



⟦When two particles are far away, the gravitational force between them vanishes, but they are still attracted by the QCD force — this explains why we never observe free quarks. At sufficiently small separations, the gravitational force is larger than the QCD force — this explains why recourse has to be made to string theory in order to quantize gravity.⟧

b. The work done is

$$\int_{x_i}^{x_f} F_Q(x)\,dx = -F_\infty \int_{x_i}^{x_f} \left(1 + \frac{\Lambda}{x}\right) dx = -F_\infty \left[x + \Lambda \ln x\right]_{x_i}^{x_f} = -F_\infty \left[x_f - x_i + \Lambda \ln(x_f/x_i)\right].$$