Oberlin College Physics 110, Fall 2011

Model Solutions to Assignment 6

HRW problem 15-4: *Car on a spring*

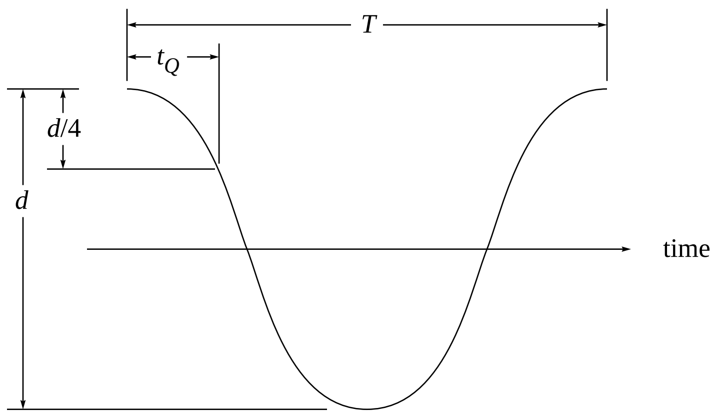
a. Use $\omega = 2\pi f = \sqrt{\frac{k}{m}}$ with $f = 3.00$ Hz and $m = 1450$ kg/4:

$$k = m(2\pi f)^2 = 1.29 \times 10^5 \text{ N/m}.$$

b. Increase m by $(5 \times 73.0 \text{ kg})/4$:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 2.68 \text{ Hz}.$$

HRW problem 15-18: *Tides*



Let's call the desired time t_Q ("time to quarter").
The height of the ocean surface is (with suitable time origin) $x(t) = A\cos(\omega t)$.
I want the height of the ocean surface when $x = A/2$, i.e. $\frac{1}{2} = \cos(\omega t_Q)$.
This comes when $\omega t_Q = 60° = \pi/3$.
But $\omega = 2\pi/T$, so $t_Q = T/6$.
Thus $t_Q = (12.5 \text{ h})/6 = 2.1$ h.

Additional problem 63: *Pendulum motion*
The period of a pendulum can only depend on these four quantities:

quantity	units
m	[kg]
L	[m]
g	[m/s^2]
$\theta_{\max}$	[none]

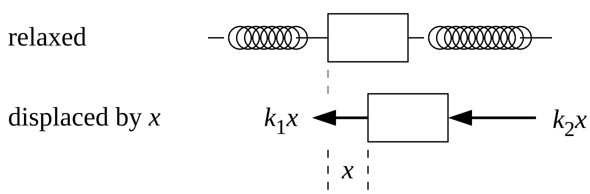
To build a period (with the units [s]) out of these four quantities, you must start with g — it's the only quantity with units involving [s]. Then you must get rid of the [m] in g, and the only way to do so is through the ratio g/L with dimensions [1/s^2]. From this, the only way to get [s] is through $\sqrt{L/g}$. Because θ_{max} is dimensionless, it can enter in any old way. But the mass m can't enter at all, because there's nothing around to cancel the [kg]. Thus dimensional analysis tells us that

$$T = f(\theta_{\text{max}})\sqrt{\frac{L}{g}}$$

and is independent of m. (Where $f(\theta_{\text{max}})$ is some unknown function.)

The physical reason for this independence is for the same reason that the acceleration of any dropped block is independent of m: greater m results in greater gravitational force, but also greater inertia, and these two effects cancel exactly.

Additional problem 68: *A block and two springs*
When the block is displaced to the right, each of the two springs pushes left. The ultimate effect is the same as one spring pushing left, where the one "effective spring" has spring constant $k_{\text{eff}} = k_1 + k_2$.



Now

$$\omega_1 = \sqrt{\frac{k_1}{m}} \qquad \omega_2 = \sqrt{\frac{k_2}{m}} \qquad \omega = \sqrt{\frac{k_{\text{eff}}}{m}}$$

so

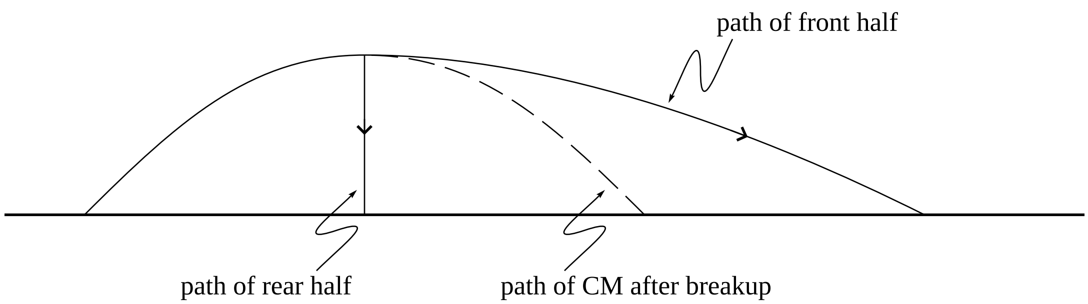
$$m\omega_1^2 = k_1 \qquad m\omega_2^2 = k_2 \qquad m\omega^2 = k_{\text{eff}}.$$

But

$$k_{\text{eff}} = k_1 + k_2 \quad \text{so} \quad \omega^2 = \omega_1^2 + \omega_2^2 \quad \text{so} \quad f^2 = f_1^2 + f_2^2 \quad \text{so} \quad f = \sqrt{f_1^2 + f_2^2}.$$

HRW problem 9–13: *The bullet that broke apart.*

Key idea: The center of mass moves like a particle subject to the net external force on the system.



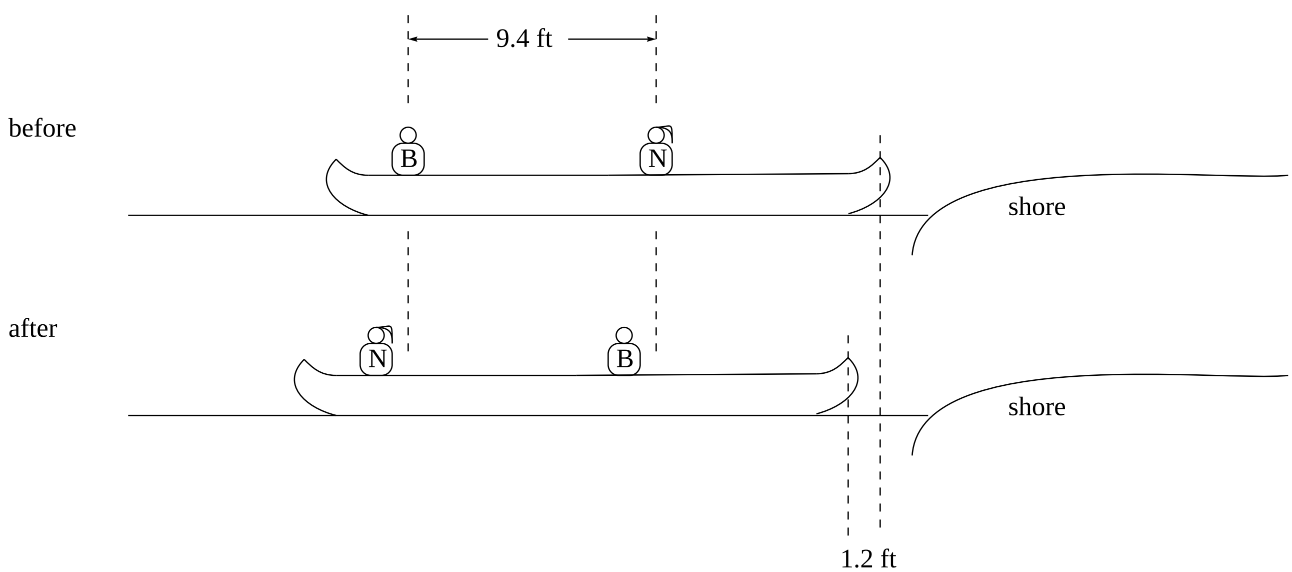
Hence range of front half = 1.5 × range of unbroken bullet.
The range of an unbroken bullet would be

$$R = \frac{v_0^2}{g} \sin(2\theta_0) = \frac{(20 \text{ m/s})^2}{9.8 \text{ m/s}^2} \sin(120°) = 35 \text{ m}.$$

So the range of the front half is 53 m.

Additional problem 85: *It happened on a moonlit night. . . .*

Key idea: Center of mass doesn't accelerate if there are no external forces.



Use m_C for the canoe's mass, x_C for the position of the canoe's center of mass, and so forth. The position of the center of mass is

$$\frac{m_B x_B + m_N x_N + m_C x_C}{m_B + m_N + m_C}.$$

This center of mass doesn't move when Ben and Natalie exchange seats, so

$$m_B \Delta x_B + m_N \Delta x_N + m_C \Delta x_C = 0$$

and

$$m_N = -\frac{m_B \Delta x_B + m_C \Delta x_C}{\Delta x_N}.$$

Because weights (W) are proportional to masses (m),

$$W_N = -\frac{W_B \Delta x_B + W_C \Delta x_C}{\Delta x_N}.$$

The figure shows very clearly that

$$\begin{aligned}
\Delta x_C &= -1.2 \text{ ft}, \\
\Delta x_B &= 9.4 \text{ ft} - 1.2 \text{ ft} = 8.2 \text{ ft}, \\
\Delta x_N &= -9.4 \text{ ft} - 1.2 \text{ ft} = -10.6 \text{ ft}
\end{aligned}$$

which, combined with $W_B = 180$ lb and $W_C = 65$ lb, gives

$$W_N = 130 \text{ lb}.$$

We have assumed that there are no external forces: Certainly we can ignore wind and water currents, because the surface of the lake is "glass-smooth." Whether we can ignore friction between the bottom of the boat and the lake water is a different question... probably okay since the kiss is quick rather than lingering.

Additional problem 87: *A girl, a sled, and an ice-covered lake, part II.*
Regardless of what the force the girl exerts on the sled, the sled exerts an opposite force on the girl, so that force cannot alter the motion of the center of mass. The girl and sled will come together at the center of mass, namely 2.6 m from the girl, for all the forces listed and for all the forces that aren't listed, too.