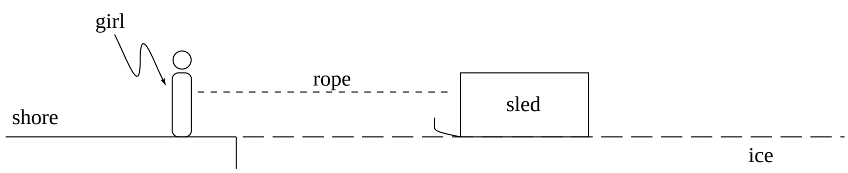
Oberlin College Physics 110, Fall 2011

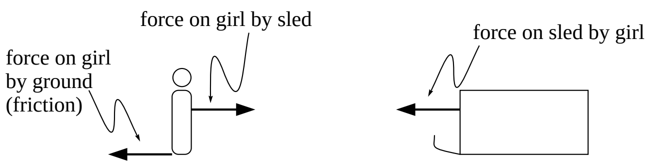
Model Solutions to Assignment 4

Additional problem 56: *A girl, a sled, and an ice-covered lake*

geometry diagram:



free body diagrams:

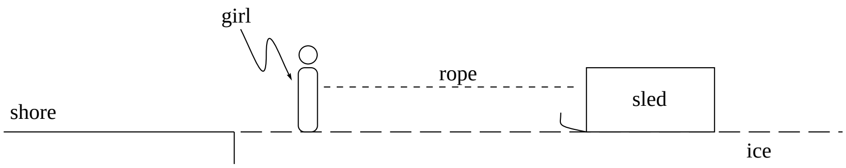


a. The only force on the sled is the leftward force due to the girl. (To be absolutely precise, the leftward force on the sled is the force due to the rope, not due to the girl. However we have seen that when the mass of the rope is negligible, then the force on the sled due to the rope is nearly equal to the force on the rope due to the girl. We say that the rope transmits the force without change. Similarly for the rightward force on the girl due to the sled.) The acceleration of the sled is

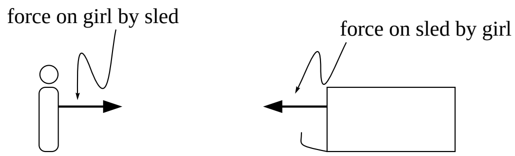
$$a_S = \frac{F_{S,\text{net}}}{m_S} = \frac{5.2 \text{ N}}{8.4 \text{ kg}} = 0.62 \text{ m/s}^2.$$

b. The two forces on the girl are the rightward force due to the sled (actually due to the rope plus sled), and the leftward force of friction due to the ground. These two forces balance so that the net force vanishes whence the acceleration is zero.

geometry diagram:



free body diagrams:



c. Same as part (a).

d. By Newton's third law, the force by the girl on the sled is equal and opposite to the force on the sled by the girl. Thus

$$a_G = \frac{F_{G,\text{net}}}{m_G} = \frac{5.2 \text{ N}}{40 \text{ kg}} = 0.13 \text{ m/s}^2.$$

e. At any instant of time, the distances traveled by the girl and the sled from their starting points are

$$x_G = \tfrac{1}{2}a_G t^2 \quad \text{and} \quad x_S = \tfrac{1}{2}a_S t^2,$$

so

$$\frac{x_S}{x_G} = \frac{a_S}{a_G} = \frac{m_G}{m_S}.$$

This makes sense: the high-mass (more sluggish, more inert) girl moves less than the low-mass sled does — but if the two were equally massive, then by symmetry they would move equal amounts.

In particular, we want to know x_G when the girl and sled touch, that is, when $x_G + x_S = 15$ m. This is when:

$$\begin{aligned}
x_G + x_S &= 15 \text{ m} \\
x_G + \frac{m_G}{m_S}x_G &= 15 \text{ m} \\
x_G\left[1 + \frac{m_G}{m_S}\right] &= 15 \text{ m} \\
x_G\left[\frac{m_S + m_G}{m_S}\right] &= 15 \text{ m} \\
x_G &= \left[\frac{m_S}{m_S + m_G}\right](15 \text{ m}) \\
x_G &= \left[\frac{8.4 \text{ kg}}{48 \text{ kg}}\right](15 \text{ m}) = 2.6 \text{ m.}
\end{aligned}$$

As expected, the low-mass sled moves more than the high-mass girl.

f. In part (b), the two forces acting on the girl are balanced: you know they are equal and opposite because the acceleration is zero, so the forces must add up to zero. But in the forces mentioned in parts (c) and (d) are equal and opposite by Newton's third law. They are equal and opposite even when the girl accelerates. The two members of a third-law pair always act upon *different* objects.

Moral of the story: I drop a ball. The gravitational force on the ball due to the earth is equal and opposite to the gravitational force on the earth due to the ball. But the ball accelerates a whole lot more than the earth does because the ball has a much smaller mass.

The word "force", like most words, has multiple meanings. (The Oxford English Dictionary lists 53 meanings for the noun "force".) In terms of the everyday meaning of the word "force", it seems absurd that the huge, enormous earth can only muster up as much force as the tiny ball can. But in the physics meaning of the word "force" this is exactly what has to happen, in order for the accelerations to be so different. I caution you that intuition developed from the military or legal senses of the word "force" probably doesn't apply to the physics sense of the word "force".

Additional problem 58: *Monkey business*
The monkey has mass m_M, the bananas have mass m_B. The drawings are:

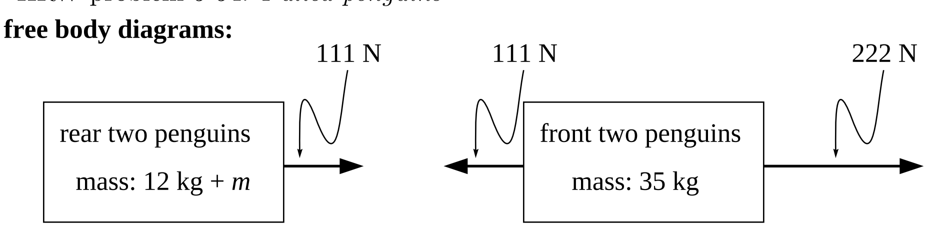
a. [8 points] The bananas don't move, so for the bananas $\sum \vec{F} = m\vec{a}$ becomes $T = m_B g$. For the monkey, $\sum \vec{F} = m\vec{a}$ becomes

$$T - m_M g = m_M a \quad \text{whence} \quad m_B g - m_M g = m_M a \quad \text{whence} \quad a = \frac{m_B - m_M}{m_M} g.$$

Given the numbers of the problem, $a = 0.50g = 4.9$ m/s^2.

b. [2 points] It is clear from our equation that if both masses are doubled, the acceleration won't change.

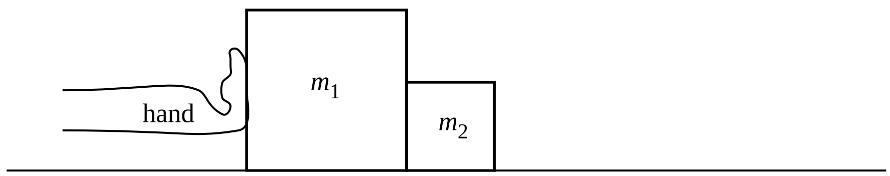
HRW problem 5-54: *Pulled penguins*



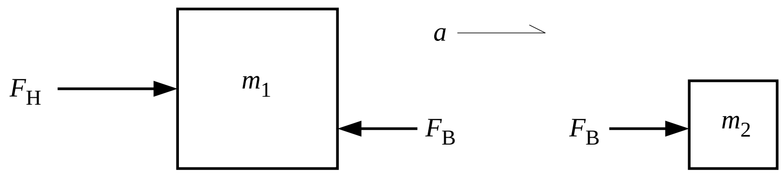
The front two penguins experience a net force of 111 N to the right, and this force results in certain acceleration. The rear two penguins experience a net force of 111 N to the right, and they also undergo the same acceleration. Thus they must have the same mass. . . the mass of the remaining penguin is 23 kg.

HRW problem 5-55: *Blocks*
Only horizontal forces are relevant, so only those are shown:

geometry diagram:



free body diagrams:



Note that both blocks undergo the same acceleration a.

For left block: $\sum F_x = m_1 a \implies F_H - F_B = m_1 a$

For right block: $\sum F_x = m_2 a \implies F_B = m_2 a$

We know m_1, m_2, and F_H. Above are two equations in two unknowns, namely a and F_B. Solving for F_B:

$$\begin{aligned} F_B &= F_H - m_1 a = F_H - m_1 \left(\frac{F_B}{m_2}\right) \\ F_B\left(1 + \frac{m_1}{m_2}\right) &= F_H \\ F_B\left(\frac{m_1 + m_2}{m_2}\right) &= F_H \end{aligned}$$

and finally

$$F_B = \left(\frac{m_2}{m_1 + m_2}\right) F_H. \tag{1}$$

Using numbers:

a.

$$F_B = \left(\frac{1.2 \text{ kg}}{3.5 \text{ kg}}\right)(3.2 \text{ N}) = 1.1 \text{ N}.$$

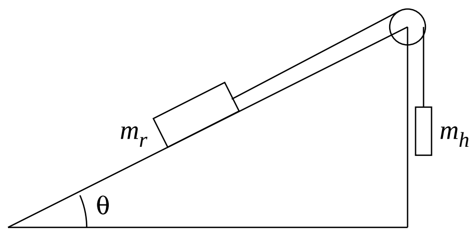
b. This situation just swaps m_1 and m_2, so now $F_B = 2.1$ N.

c. Result (1) says that F_B diminishes as m_2 shrinks: From $F_B = F_H$ if $m_1 = 0$ down to $F_B = 0$ if $m_2 = 0$.

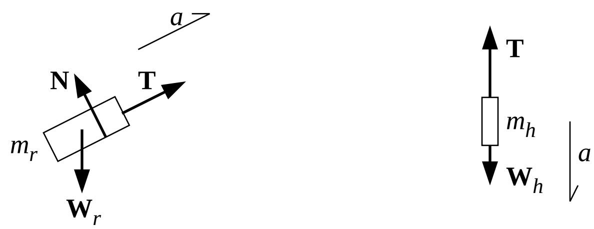
HRW problem 5-57: *Wedge*

As usual, we solve this problem first with symbols (m_r, m_h, and θ), and only then plug in numbers. This is because (1) it is easier to do algebra with symbols than numbers and (2) we will be able to "read the equation" to check it for reasonableness.

geometry diagram:



free body diagrams:



(Note: we could have chosen the two positive acceleration directions pointing the other way... this would have just resulted in a final acceleration of the opposite sign.) Apply $\sum \vec{F} = m\vec{a}$ to both blocks:

	$\sum F = ma$	
Forces on hanging block	$m_h g - T = m_h a$	(1)
Forces on ramp block, parallel to surface	$T - m_r g \sin\theta = m_r a$	(2)
Forces on ramp block, perpendicular to surface	$N - m_r g \cos\theta = 0$	(3)

Equations (1) and (2) are two equations for the two unknowns T and a. Solve them simultaneously to find

$$a = \frac{m_h - m_r \sin\theta}{m_h + m_r} g,$$

$$T = \frac{m_h m_r}{m_h + m_r}(1 + \sin\theta) g.$$

Now, do these equations make sense?

The dimensions are correct. Check.

When $m_r = 0$, these equations give $a = g$ and $T = 0$. Check.

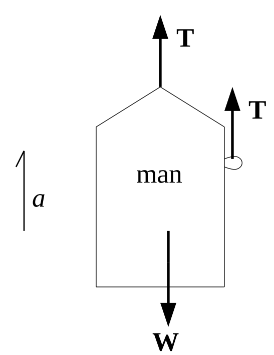
When $\theta = 90°$ this is exactly the situation of the additional problem on "sliding salami", and these equations give exactly the same answers. Check.

If $m_h \gg m_r$, the acceleration is positive; if $m_r \gg m_h$, the acceleration is negative. Check.

I can't think of any more circumstances in which I have any intuition. Can you?

Plugging in the numbers given, and using significant figures properly, we find that for the situation of this problem: (a) $a = 0.736$ m/s^2; (b) the hanging block accelerates downward; (c) $T = 20.9$ N.

HRW problem 5-58: *Window washer*

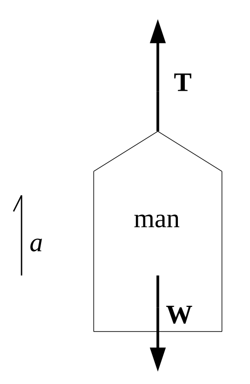


The equation $\sum \vec{F} = m\vec{a}$ becomes

$$2T - mg = ma, \quad \text{so} \quad T = \tfrac{1}{2}m(a+g).$$

(a) When $a = 0$, $T = \frac{1}{2}(95.0 \text{ kg})(9.81 \text{ m/s}^2) = 466$ N.

(b) When $a = 1.30$ m/s^2, $T = \frac{1}{2}(95.0 \text{ kg})(1.30 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 528$ N.



In this case the equation $\sum \vec{F} = m\vec{a}$ becomes

$$T - mg = ma, \quad \text{so} \quad T = m(a+g).$$

(c) When $a = 0$, $T = (95.0 \text{ kg})(9.81 \text{ m/s}^2) = 932$ N.

(d) When $a = 1.30 \text{ m/s}^2$, $T = (95.0 \text{ kg})(1.30 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 1060$ N.

Discussion: Suppose we want the cab to rise at constant speed. The man in the cab accomplishes this by applying a force of 466 N. His coworker on the ground needs to apply twice as much force! It seems that the man in the cab is getting something for nothing.

However, consider this aspect: When the coworker on the ground pulls two meters of rope through his hands, the man in the cab rises by two meters. But when the man in the cab pulls two meters of rope, the cab rises by only one meter. (One meter is eliminated between the cab and the pulley, the other meter is eliminated between the pulley and the hand.)

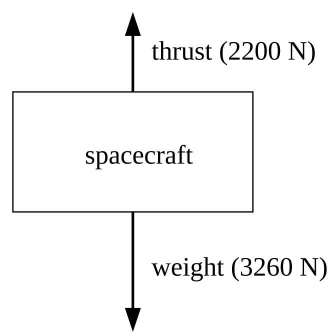
So the man in the cab pulls with half the force, but has to pull twice as much rope in order to rise the same distance. We will return to this issue when we discuss work.

Problem 5-88: *Landing on Callisto*

a. The weight is 3260 N.

b. When the thrust is 2200 N, the net force is 1060 N (downward), so the mass of the spacecraft is

$$m = \frac{F_{\text{net}}}{a} = \frac{1060 \text{ N}}{0.39 \text{ m/s}^2} = 2700 \text{ kg}.$$

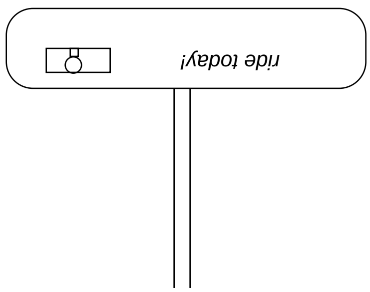


c. The acceleration of gravity is

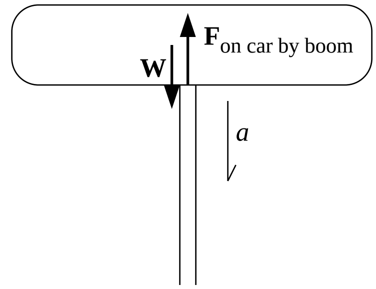
$$\frac{\text{weight}}{\text{mass}} = \frac{3260 \text{ N}}{2700 \text{ kg}} = 1.2 \text{ m/s}^2.$$

HRW problem 6-52: *Amusement park*

geometry diagram:



free body diagram:



The equation $\sum \vec{F} = m\vec{a}$ becomes (taking the positive direction to be downward)

$$W - F_{\text{on car by boom}} = ma = m\frac{v^2}{r}.$$

So

$$F_{\text{on car by boom}} = W - m\frac{v^2}{r} = W\left(1 - \frac{v^2}{rg}\right).$$

For the two cases given:

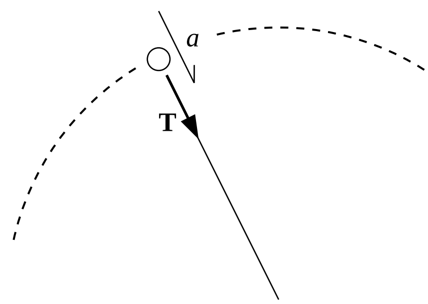
a. If $v = 5.0$ m/s, then $F = +3.7$ kN (that is, the force points upward, as shown in the diagram).

b. If $v = 12$ m/s, then $F = -2.3$ kN. (The negative sign means that the force points downward.)

It makes sense that at low speeds, the force should point upward, because at zero speed the force is of course upward. As the speed increases, the boom needs to hold on to the car lest it fly away.

HRW problem 6-57: *Orbiting puck*

free body diagram:



The cylinder will remain at rest when the string tension $\vec{T}$ supplies all the needed centripetal acceleration v^2/r. This occurs when

$$T = Mg = m\frac{v^2}{r} \quad \text{or} \quad v = \sqrt{\frac{M}{m}rg}.$$

(With a lower puck speed, the cylinder drops. With a higher puck speed, it rises.)

HRW problem 6-54: *Amusement park design*

The net force on the passenger must be

$$F = \frac{mv^2}{r}.$$

a. If there is a small change dr in the radius, with no change in v, then the force will change by about

$$dF = \frac{dF}{dr}dr = -\frac{mv^2}{r^2}dr.$$

b. If there is a small change dv in the speed, with no change in r, then the force will change by about

$$dF = \frac{dF}{dv}dv = \frac{2mv}{r}dv.$$

c. If there is a small change dT in the period

$$T = \frac{2\pi r}{v},$$

with no change in r, then the associated change in velocity is

$$dv = \frac{dv}{dT}dT = -\frac{2\pi r}{T^2}dT.$$

Using the result of part (b), this changes the force by

$$dF = \frac{2mv}{r}dv = -\frac{2mv}{r}\frac{2\pi r}{T^2}dT = -\frac{8\pi^2 mr}{T^3}dT.$$