Oberlin College Physics 110, Fall 2011

Model Solutions to Assignment 1

Additional problem 1: *Weighting light things with a heavy standard*

Make a blob of clay that balances the standard.
Cut the clay about in half, then place each portion on the two sides of the balance.
Adjust the two portions until they balance.
Now each portion has the mass of one-half the standard.

Execute this process with the kilogram standard to produce a half-kilogram standard.
Execute this process with the half-kilogram standard to produce a quarter-kilogram standard.
Continue until you have a collection of standards down to as small as the desired accuracy demands.

Now balance the low-mass apple with these low-mass standards.

Additional problem 2: *Significant figures*

(a) $3.4 \times 7.954 = 27$ (two significant figures)
(b) $99.3 + 98.7 = 198.0$ (the "tenth" place is significant)
(c) $99.3 - 98.7 = 0.6$ (the "tenth" place is significant)
(d) (Use the procedure described in workshop.)

$$\begin{aligned} \cos(3.200\ldots^\circ) &= 0.99844076\ldots \\ \cos(3.300\ldots^\circ) &= 0.99834181\ldots \\ \text{hence} & \\ \cos(3.2^\circ) &= 0.9984 \end{aligned}$$

(e) 5×2.134 meters $= 10.67$ meters (The five is exact.)

Additional problem 3: *A new law of nature?*

If I look at the numerical values alone, expressed in meters per second ($c = 299,792,458$ m/sec), I have

v_s	$\frac{1}{2}\sqrt[3]{c}$
340.29	334.64

This is not in perfect agreement, but it's pretty close…only 1.2% error.

However if I look at the numerical values expressed in kilometers per second ($c = 299,792.458$ km/sec), then I have

v_s	$\frac{1}{2}\sqrt[3]{c}$
0.34029	33.464

These two numbers are quite far apart. What's going on?

Remember that the *quantity* c consists of the *numerical value* 299, 792, 458 times the *unit* m/sec. The two boxes above considered only the numerical value and ignored the unit which, remember, is like taking the expression "$6x$" and treating it as "6". If I evaluate the quantities (rather than just the numerical values) I find

v_s	$\frac{1}{2}\sqrt[3]{c}$
340.29 m/sec	334.64 $\text{m}^{1/3}/\text{sec}^{1/3}$

The two sides of the equation are dimensionally inconsistent, so the equation *can't* be correct — or even meaningful. This flaw reveals itself numerically through the fact that the numerical relation nearly holds using one set of units but fails miserably for a different set. The close numerical agreement when the units are m/sec is pure coincidence.

Additional problem 5: *A Doll's House*
The real house has volume

$$(12.0 \text{ m}) \times (20.0 \text{ m}) \times (6.00 \text{ m}) + \tfrac{1}{2}(12.0 \text{ m}) \times (20.0 \text{ m}) \times (3.00 \text{ m}) = 1,800 \text{ m}^3.$$

(Technically, this should be written as 1.80×10^3 m^3, because the result has three significant digits.)
(a) For the doll house, each length would decrease by a factor of 1/12, so the volume would decrease by a factor of $(1/12)^3 = 1/1728$ to 1.04 m^3.
(b) For the miniature house, the volume would again decrease by a factor of 1/1728 to 6.03×10^{-4} m^3.

Additional problem 15: *How long is a lecture?*
(a) How many minutes in a century?

$$1 \text{ century} = 1 \text{ century} \left(\frac{100 \text{ year}}{1 \text{ century}}\right)\left(\frac{365.25 \text{ day}}{1 \text{ year}}\right)\left(\frac{24 \text{ hour}}{1 \text{ day}}\right)\left(\frac{60 \text{ minute}}{1 \text{ hour}}\right) = 52.596 \times 10^6 \text{ minute}$$

So a microcentury is 52.596 minutes, nearly equal to a standard class lecture. (Note: If you used the approximation of 365 days per year, you found that a microcentury was 52.560 minutes.)
(b) A microcentury differs from a standard 50 minute lecture by

$$\left|\frac{50 \text{ minute} - 52.596 \text{ minute}}{50 \text{ minute}}\right| \times 100\% = 5\%.$$

Additional problem 16: *To see a world in a grain of sand...*
This is a problem of "order of magnitude estimation". It's silly to solve this problem to high accuracy, because a grain of sand *isn't* exactly a sphere and its radius *isn't* exactly 50 μm.

One grain of sand has:

radius: 50 $\mu\text{m} = 5 \times 10^{-5}$ m
surface area: $4\pi r^2 \approx 4 \times 3 \times (5 \times 10^{-5}\text{ m})^2 = 3 \times 10^{-8}\text{ m}^2$
volume: $(4/3)\pi r^3 \approx 4 \times (5 \times 10^{-5}\text{ m})^3 = 5 \times 10^{-13}\text{ m}^3$
mass: [density]×(volume) = $[(2600\text{ kg})/(1\text{ m}^3)]\times(5 \times 10^{-13}\text{ m}^3) = 1.3 \times 10^{-10}$ kg

So, how many grains have a surface area of 6 m^2?

$$\text{grains needed} = \frac{\text{surface area needed}}{\text{surface area/grain}} = \frac{6\text{ m}^2}{3 \times 10^{-8}\text{ m}^2} = 2 \times 10^8.$$

What is the mass of this many grains?

$$\text{mass} = (\text{number of grains}) \times (\text{mass per grain}) = (2 \times 10^8) \times (1.3 \times 10^{-9}\text{ kg}) = 0.3\text{ kg}.$$

So a block of 2600 kg of solid sand has the same surface area as a mere quarter kilogram of sand grains!

Additional problem 17: *Threads and sheets*

a.
$$\text{linear density } = \lambda = \frac{12\text{ grams}}{1500\text{ m}} = 0.0080\text{ grams/m}$$

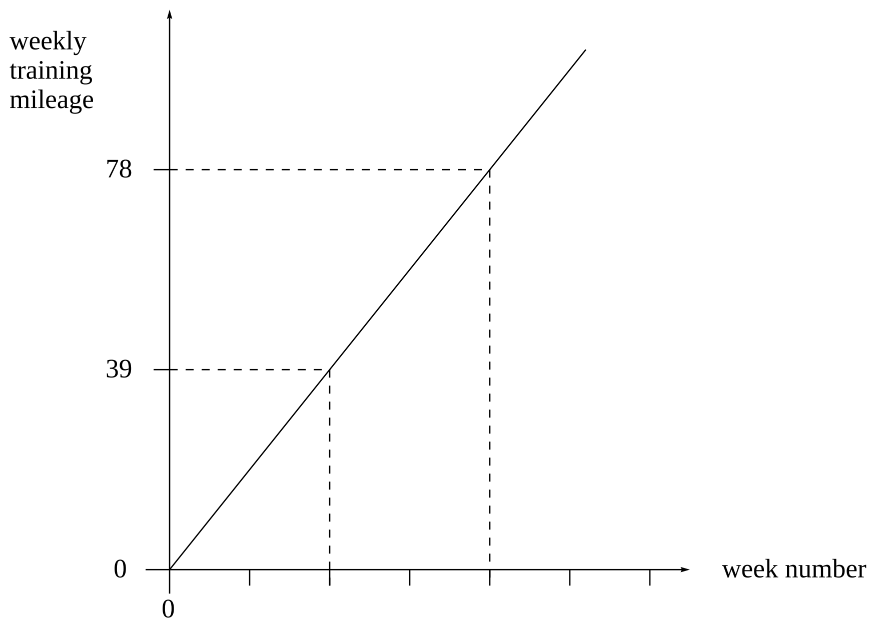
b. There are both horizontal and vertical threads (the "warp" and the "woof"). A square meter of sheet contains 8500 horizontal threads and 8500 vertical threads, each thread with a mass of 0.0080 gram. Thus a square meter has a mass of $2 \times 8500 \times 0.0080$ g = 0.14 kg, so

$$\text{surface density } = \sigma = 0.14\text{ kg/m}^2.$$

Additional problem 18: *Marathon training*

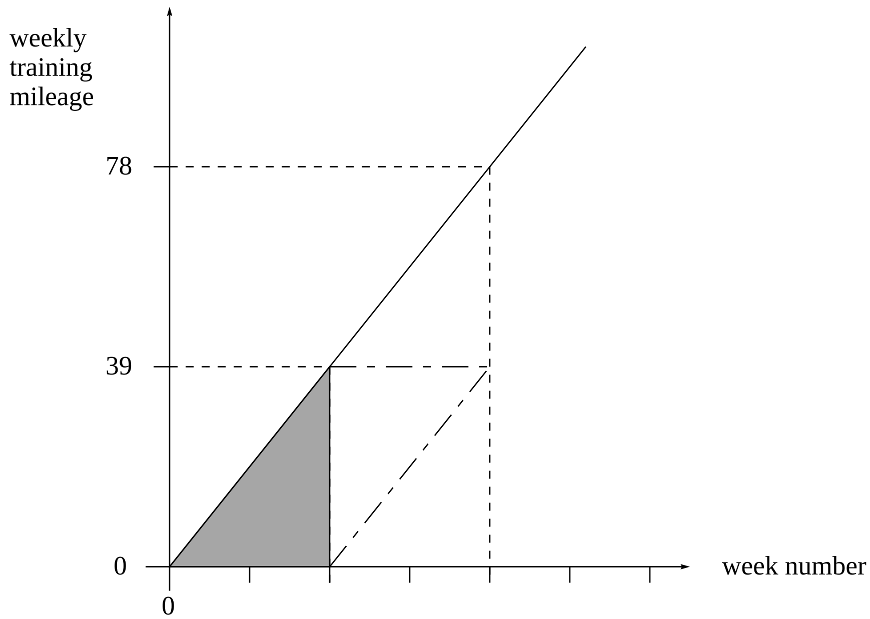
There are many ways to solve this problem ("brute force" addition, sum of a linear progression, etc.). I'll show you the way that provides the most insight, but any correct solution will earn full credit.

(a) The weekly training mileage increases by a the same amount each week.



It will take a certain amount of time to reach the threshold for a half marathon (namely 3×13 miles = 39 miles) and twice as long to reach the threshold for a marathon (78 miles). (Note that I haven't labeled the ticks on the horizontal "week number" axis. I don't need to. The question asks not for the number of weeks required, but for how the number of weeks required scales with the distance of the race.)

(b) The cumulative training mileage is proportional to the area under the curve. For example, the cumulative training mileage for the half marathon is represented by the grayed-out area in the graph below.



Now, can you see from the graph that the area of the triangle stretching up to 78 miles is four times the area of the out-out triangle? The dot-dash lines are supposed to help you. Once you see this geometrical fact, it's obvious that the cumulative training mileage for the marathon is four times the cumulative training mileage for the half-marathon.

(c) Training for a "sesquimarathon" requires three times the time and nine times the cumulative training mileage as training for the half-marathon. In general, the training time scaled linearly with the race distance, whereas the cumulative training mileage scales with the square of the race distance.