Oberlin College Physics 110, Fall 2011

Model Solutions to Sample Final Exam

Additional problem 27: *Cannon shot*

Because I'm concerned about speeds and distances, but not times, the central equation will likely be

$$v^2 = v_0^2 + 2a_0(x - x_0).$$

In our case $v_0 = 0$, $x - x_0 = L$, the length of the cannon, so

$$a_0 = \frac{v^2}{2L}.$$

To find the time, use

$$v = v_0 + a_0 t$$

or

$$T = \frac{v}{a_0} = \frac{2L}{v}.$$

Plugging in the given numbers results in a time of 0.6600 s. (Note four significant figures.)

Additional problem 76: *Spring gun*

Solved in the notes.

Additional problem 90: *Train latch*

Let M_f and M_c represent the masses of the freight car and the caboose. Let v_i represent the initial velocity of the freight car and v_f represent the final velocity of the latched combination.

Momentum conservation: $M_f v_i = (M_f + M_c) v_f$.

Kinetic energy loss: $0.66(\frac{1}{2} M_f v_i^2) = \frac{1}{2}(M_f + M_c) v_f^2$.

Square the first equation, and divide that squared equation by the second equation to obtain

$$\frac{1}{0.66} M_f = M_f + M_c \quad \text{or} \quad M_c = \frac{0.37}{0.66} M_f.$$

Plugging in the given weight of the freight car, the caboose has weight 24 tons.

HRW problem 9-17: *A dog on a boat*

The location of the center of mass is

$$x_{cm} = \frac{x_B m_B + x_D m_D}{m_B + m_D}$$

where x_B is the location of the CM of the boat, m_B the mass of the boat, and similarly for the dog.

There are no external forces during the dog's walk, so $\Delta x_{cm} = 0$, whence

$$\Delta x_B m_B = -\Delta x_D m_D.$$

Now, $\Delta x_D = -2.4\text{ m} + \Delta x_B$ so

$$\Delta x_B = (2.4\text{ m})\left(\frac{m_B}{m_B + m_D}\right) = 0.48\text{ m}.$$

So the distance from the dog to shore is

$$6.1 \text{ m} - 2.4 \text{ m} + 0.48 \text{ m} = 4.2 \text{ m}.$$

Relativity problem 2: *Muon lifetime*
Classically, without time dilation:

$$\text{distance traveled} = \text{speed} \times \text{time} = (0.83\,c) \times (2.2\ \mu\text{s}) = 550 \text{ m}$$

Correctly, with time dilation:

$T_0 = $ time ticked off by muon between production and decay $= 2.2\ \mu$s

$T = $ time elapsed in lab frame between production and decay $= \dfrac{T_0}{\sqrt{1-(V/c)^2}} = \dfrac{2.2\ \mu\text{s}}{\sqrt{1-(0.83)^2}} = 3.9\ \mu$s

distance traveled in lab frame = speed in lab frame × time elapsed in lab frame = $(0.83\,c) \times (3.9\ \mu\text{s}) = 980$ m

Relativity problem 8: *Time travel*
Ivan has aged $T_0 = 2$ years whereas time $T = 12$ years has elapsed, so

$$\begin{aligned} T &= T_0/\sqrt{1-(V/c)^2} \\ \sqrt{1-(V/c)^2} &= T_0/T = 1/6 \\ 1-(V/c)^2 &= 1/36 \\ (V/c)^2 &= 35/36 \\ V &= \sqrt{35/36}\,c = 0.986\,c \end{aligned}$$

Relativity problem 11: *Two events*
The Lorentz transform says that

$$\Delta t' = \frac{\Delta t - V\Delta x/c^2}{\sqrt{1-(V/c)^2}}.$$

So if $\Delta t' = 0$, we have

$$\begin{aligned} 0 &= \Delta t - V\Delta x/c^2 \\ V &= (\Delta t/\Delta x)c^2 = (6 \text{ nan}/14 \text{ ft})(1 \text{ ft/nan})c = \tfrac{3}{7}c. \end{aligned}$$

Relativity problem 18: *Relativistic energy: a new proposal*
If this proposal were correct, then the non-relativistic limit of energy would be

$$E = \frac{mc^2}{\sqrt{1-(v/c)^4}} \approx mc^2\left[1 - \tfrac{1}{2}\left(-(v/c)^4\right)\right] = mc^2 + \tfrac{1}{2}mv^4/c^2.$$

That is, classical kinetic energy would be, not $\frac{1}{2}mv^2$, but $\frac{1}{2}mv^4/c^2$. Clearly wrong.