Units in electromagnetism

Almost all textbooks on electricity and magnetism (including Griffiths's book) use the same set of units — the so-called rationalized or Giorgi units. These have the advantage of common use. On the other hand there are all sorts of "ϵ_0"s and "μ_0"s to memorize. Could anyone think of a system that doesn't have all this junk to memorize?

Yes, Carl Friedrich Gauss could. This problem describes the Gaussian system of units. [In working this problem, keep in mind the distinction between "dimensions" (like length, time, and charge) and "units" (like meters, seconds, and coulombs).]

a. In the Gaussian system, the measure of charge is

$$\tilde{q} = \frac{q}{\sqrt{4\pi\epsilon_0}}.$$

Write down Coulomb's law in the Gaussian system. Show that in this system, the dimensions of $\tilde{q}$ are

$$[\text{length}]^{3/2}[\text{mass}]^{1/2}[\text{time}]^{-1}.$$

There is no need, in this system, for a unit of charge like the coulomb, which is independent of the units of mass, length, and time.

b. The electric field in the Gaussian system is given by

$$\tilde{\vec{E}} = \frac{\vec{F}}{\tilde{q}}.$$

How is this measure of electric field ($\tilde{\vec{E}}$) related to the standard (Giorgi) field ($\vec{E}$)? What are the dimensions of $\tilde{\vec{E}}$?

c. The magnetic field in the Gaussian system is given by

$$\tilde{\vec{B}} = \vec{B}\sqrt{\frac{4\pi}{\mu_0}}.$$

What are the dimensions of $\tilde{\vec{B}}$ and how do they compare to the dimensions of $\tilde{\vec{E}}$?

d. In the Giorgi system, the Lorentz force law is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

What is the Lorentz force law expressed in the Gaussian system? Recall that $c = 1/\sqrt{\epsilon_0\mu_0}$.

e. In the Giorgi system, the Biot-Savart law is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}.$$

What is the Biot-Savart law in the Gaussian system?

f. In the Gaussian system, of course,

$$\tilde{\rho} = \frac{\rho}{\sqrt{4\pi\epsilon_0}} \quad \text{and} \quad \tilde{\vec{J}} = \frac{\vec{J}}{\sqrt{4\pi\epsilon_0}}.$$

Write down the Maxwell equations in the Gaussian system.

g. [Optional.] *Circuit elements.* In the Gaussian system, what are the dimensions of capacitance? Of inductance? Of resistance?

h. [Optional.] *Magnetic moment.* In the Giorgi system, the magnetic dipole moment of a loop carrying current i around area A is defined to have magnitude $m = iA$. In the Gaussian system, it is defined to have magnitude $\tilde{m} = \tilde{i}A/c$. What is the relation between m and $\tilde{m}$?

i. [Optional.] *Magnetic materials.* I said in the introduction that the Giorgi system has "the advantage of common use". But in one discipline, namely magnetic materials, it's actually more common to use the Gaussian system. In the Giorgi system, magnetic intensity $\vec{H}$ is defined by $\vec{H} \equiv \vec{B}/\mu_0 - \vec{M}$, where the "magnetic polarization" $\vec{M}$ is the magnetic dipole moment per volume of a magnetic material. In the Gaussian system, $\tilde{\vec{H}} \equiv \vec{H}\sqrt{4\pi\mu_0}$. What is the relation between $\tilde{\vec{H}}$, $\tilde{\vec{B}}$, and $\tilde{\vec{M}}$?

Conventionally, the Giorgi system is used along with meters, kilograms, and seconds while the Gaussian system is used along with centimeters, grams, and seconds. For that reason the Giorgi system is often called "MKS" or "SI", while the Gaussian system is often called "cgs". But this is mere convention… either electromagnetic system can be used alongside any system of measuring length, mass, and time (even feet, slugs, and minutes!).

⟦If you look at a book or paper that uses the Gaussian system, you will not find a bunch of quantities written with tildes like $\tilde{q}$. Instead, these books simply drop the tildes and state that "this book uses the Gaussian system of units". Or, they might use the Gaussian system and leave it to the reader to figure out from context that the Gaussian system is in use.⟧

⟦There are other electromagnetic unit systems: In the Lorentz-Heaviside system, like the Gaussian system, there are no "ϵ_0"s or "μ_0"s. But in the Gaussian system there are "4π"s in the Maxwell equations. In the Lorentz-Heaviside system there are no "4π"s in the Maxwell equations: they instead show up in the Coulomb and Biot-Savart laws. The Lorentz-Heaviside system is often used in formal or mathematical treatments of field theory, such as treating electromagnetism not in three-dimensional space, but in space of arbitrary dimensionality.⟧