# Radiation

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We have seen that electromagnetic fields can exist far from any charge or current, where the electric field is produced by changing magnetic field, and the magnetic field is produced by changing electric field.

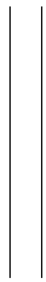
But how does such a field configuration get started? Here's one way.

Suppose an infinite neutral slab of metal moves in toward this sheet of paper. Then the metal hits an obstruction (perhaps the tabletop on which this sheet of paper rests). The nuclei of the metal stop moving at once, but the electrons, being not so well bound as the nuclei, continue moving in for a short time, and then they stop as well.

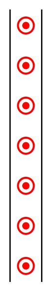
In this scenario, at first there is no net charge or current. During the short time in the middle, there is no net charge, but there is a net current outward, away from the sheet of paper. And at the end there is no net charge or current.

Side View

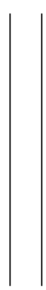
first (slab moving in)



middle (nuclei stopped, electrons moving in, current out)



end (slab stopped)

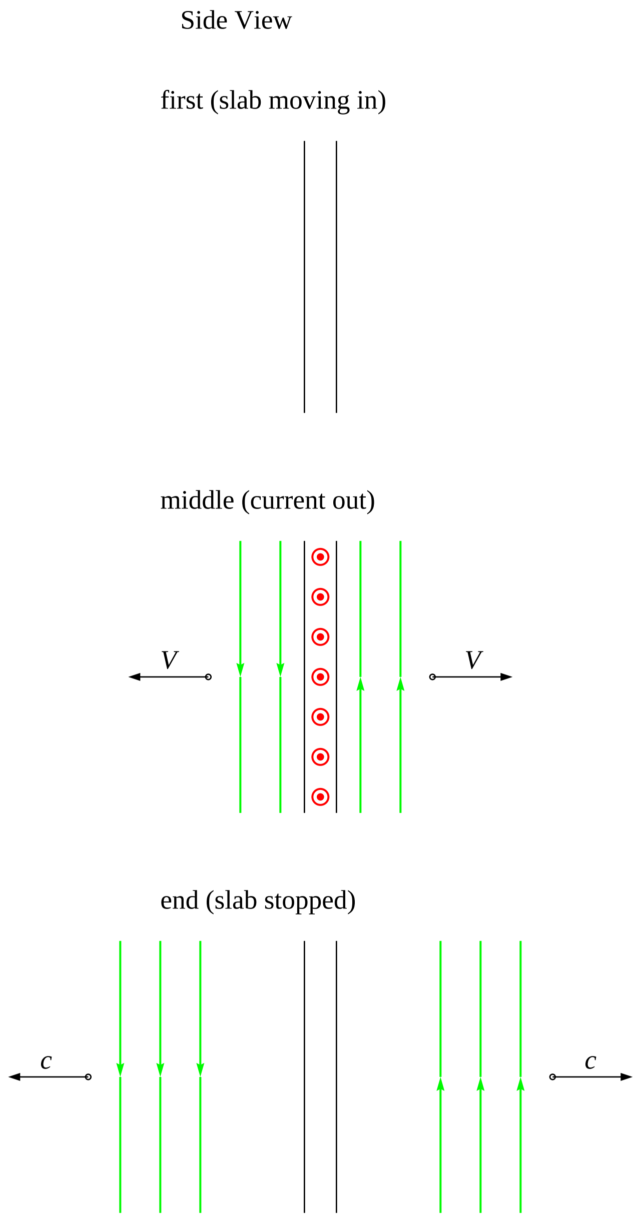


What electric and magnetic fields are produced? As usual, we use symmetry and similar arguments to find the qualitative character of the field, then use the loop rules

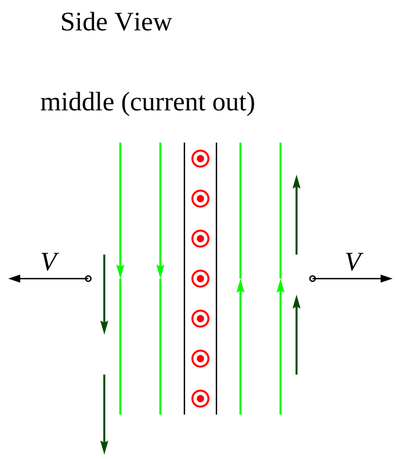
$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad \text{and} \quad \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{linked}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{1}$$

to find the quantitative magnitudes for those fields.

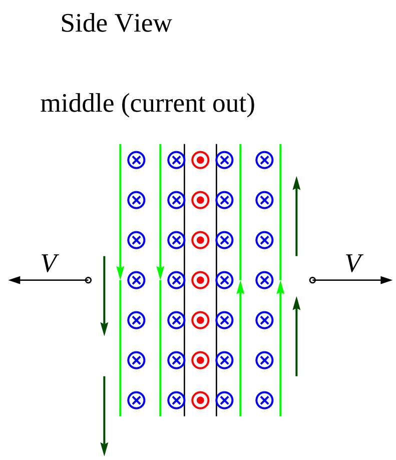
As far as qualitative character goes, there will be no fields at first. It's clear that in the middle the current will generate magnetic field, but it doesn't seem sensible that the magnetic field due to the slab stoppage will instantly extend throughout all of space. It's more reasonable to suppose that this field expands into field-free space at some speed $V$. Then at the end, when there are no longer charges or currents, there will not be any field generated by the metal slab, but the two slabs of magnetic field initially set up by the current in the metal will continue in the light-like manner that we've seen before.



Now, of course, as the magnetic field expands into field-free space there is a front of $d\vec{B}/dt$.



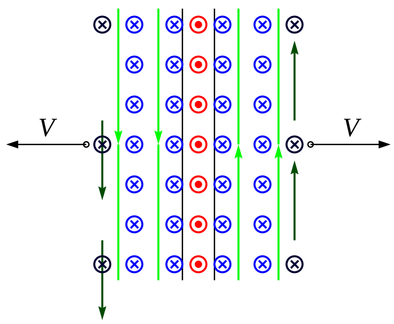

That changing magnetic field produces electric field according to Faraday's law and the "anti-right-hand rule."



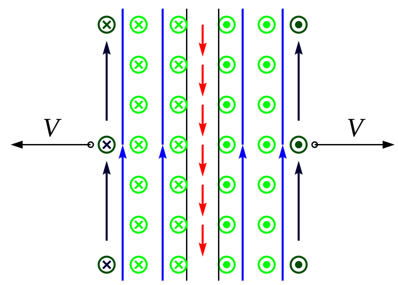
And as the electric field expands into field-free space there is a front of $d\vec{E}/dt$.

Side View

middle (current out)



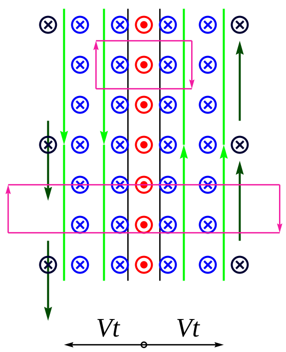

Top View



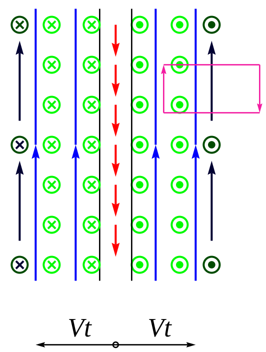

Now that we have a good qualitative picture of the fields, it's time to integrate over the loops to find their values. Suppose that the current per length is $\lambda$, and that we take a snapshot at time $t$ after the current burst has started.

Side View

middle (current out)




Top View




Apply the Ampere-Maxwell law to the top purple loop on the "Side View":

$$\begin{aligned} \oint \vec{B}\cdot d\vec{\ell} &= \mu_0 I_{\text{linked}} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} \\ 2|\vec{B}|(\text{height of loop}) &= \mu_0\lambda(\text{height of loop}) + 0 \\ |\vec{B}| &= \tfrac{1}{2}\mu_0\lambda \end{aligned} \tag{2}$$

And apply it to the bottom purple loop on the "Side View":

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{linked}} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$

$$\begin{aligned} 0 &= \mu_0\lambda(\text{height of loop}) + \mu_0\epsilon_0 \frac{d}{dt}\left[-|\vec{E}|(2Vt)(\text{height of loop})\right] \\ |\vec{E}|V &= \frac{1}{2\epsilon_0}\lambda \end{aligned} \tag{3}$$

Meanwhile, apply Faraday's Law to the purple loop on the "Top View":

$$\begin{aligned} \oint \vec{E}\cdot d\vec{\ell} &= -\frac{d\Phi_B}{dt} \\ |\vec{E}|(\text{height of loop}) &= |\vec{B}|V(\text{height of loop}) \\ |\vec{E}| &= |\vec{B}|V \end{aligned} \tag{4}$$

Putting these results together, we find

$$V = c \quad \text{and} \quad |\vec{E}| = |\vec{B}|c \tag{5}$$

just as for light moving far from any charge or current.

There are two take-home points. First, when light originates it doesn't start up slowly and then accelerate to cruising speed, the way a car or an airplane does. It starts off moving at the speed of light instantly. Second, the radiation doesn't happen when charges are stationary or moving steadily – it happens when charges accelerate.