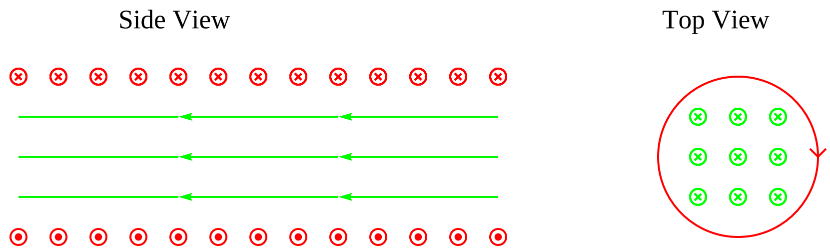
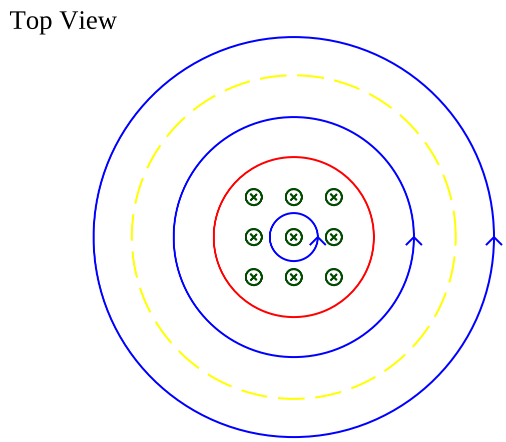
Solenoid

Griffiths, *Electrodynamics*, fourth edition, problem 7.15

Part I: Qualitative character of field. The field inside has magnitude $B(t) = n\mu_0 I(t)$, the field outside is zero. (Current shown in red, $\vec{B}$ shown in green.)



If the current increases, then B increases, and that $\frac{d\vec{B}}{dt}$ creates $\vec{E}$ according to the anti-right-hand rule. ($\frac{d\vec{B}}{dt}$ shown in dirty green, $\vec{E}$ shown in blue.)



Part II: Magnitude of field. To find the magnitude of that $\vec{E}$, apply Faraday's Law to the dashed yellow loop of radius s:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{\ell} &= \frac{d\Phi_B}{dt} \\ E(2\pi s) &= n\mu_0 \frac{I(t)}{dt} \times \text{(area within both loop and solenoid)} \end{aligned}$$

Now,

$$\text{area within both loop and solenoid} = \begin{cases} \pi a^2 & \text{for } s > a \text{ (outside solenoid)} \\ \pi s^2 & \text{for } s < a \text{ (inside solenoid)} \end{cases}$$

The upshot is that outside the solenoid,

$$E(s) = \frac{1}{2} n\mu_0 \frac{dI}{dt} \frac{a^2}{s},$$

whereas inside the solenoid,

$$E(s) = \frac{1}{2} n\mu_0 \frac{dI}{dt} s.$$