The ladybug flies away

Dan Styer, Oberlin College, 25 February 2021

A ladybug flies from the rear windshield of a car to the front windshield of a car. If we use

w_b for the speed of the ladybug relative to the Earth,
v_b for the speed of the ladybug relative to the car, and
V for the speed of the car relative to the Earth,

then common sense tells us that

$$w_b = v_b + V.$$

But in truth (that is in relativity) these quantities are related through

$$w_b = \frac{v_b + V}{1 + v_b V/c^2}.$$

If v_b and V are both positive, then the true w_b is *less* than the result of the common-sense formula. This is why speed additions that common sense tells you should be greater than c are in fact less than c.

Chapter 11, "Speed Addition", of the book *Relativity for the Questioning Mind* derives this true formula, and shows how it grows out of time dilation, length contraction, and the relativity of synchronization. But the derivation is technical and doesn't give any real intuition into what's going on. Why, for example, is the true w_b *smaller* than the common-sense result?

Let's picture the ladybug flying in the frame of the car. It moves the length of the car's cab, and it flaps its wings quickly: flapflapflapflapflapflap. . . .

Now picture the same flight in the frame of the Earth. Because the moving car cab is short, the ladybug has less distance to travel from the rear windshield to the front windshield. And any moving clock ticks slowly — to give two examples of clocks, a moving mustache grows slowly, and a moving ladybug flaps slowly: flap . . . flap . . . flap . . . flap . . . flap . . . flap . . . .

As observed in the Earth's frame, the ladybug travels a lesser distance over a longer time, so naturally the added speed is less than common sense predicts.

Try this out numerically: Suppose

$$v_b = 50 \text{ miles/hour} \qquad \text{and} \qquad V = c - 10 \text{ miles/hour}.$$

If you work out the Einstein velocity addition formula you will find that

$$w_b = c - 9.999\,998\,5 \text{ miles/hour}.$$

Phrased differently, in the car's frame the ladybug travels 50 miles/hour faster than the car, but in the Earth's frame it travels 0.000 001 5 miles/hour faster than the car.