Model Solutions for the Einstein and Relativity Sample Final Exam

Short answer part

A train is 900 feet long in its own frame, and a tunnel is 1500 feet long in its own frame. Ivan sits at the very front of the train and Veronica sits at the very rear. The train speeds from west to east through the tunnel at $V = \frac{4}{5}c$ (so that $\sqrt{1-(V/c)^2} = \frac{3}{5}$).

In the train's frame: The moving tunnel is length contracted to be exactly 900 feet long [900 feet = $\frac{3}{5} \times$ (1500 feet)]. Ivan and Veronica both stick out their heads and glance up at the instant that the train exactly fits within the tunnel, so Veronica sees the west portal of the tunnel and Ivan sees the east portal of the tunnel. All of these questions deal with the situation **in the tunnel's frame.**

1. In the tunnel's frame, how long is the train?
By length contraction, the train is a shorter length 540 feet = $\frac{3}{5} \times 900$ feet.

2. In the tunnel's frame, Veronica glances before Ivan does because her clock is set ahead of his. By how much is Veronica's clock set ahead?
By the relativity of synchronization, the rear clock is set ahead by $L_0 V/c^2 = (900 \text{ feet})(\frac{4}{5}c)/c^2 = 720$ nan.

3. While Ivan's watch ticks off 720 nans, how much time elapses in the tunnel's frame? (In other words, how much time elapses between Veronica's glance and Ivan's glance?)
By time dilation, the time elapsed is longer than the time ticked off by the moving clock. The time elapsed is 1200 nan = $\frac{5}{3} \times 720$ nan.

4. During the time between glances, how far does the train move (in the tunnel's frame)?
Distance = speed × time = $\frac{4}{5}c \times 1200$ nan = 960 feet.

5. Sum your answer to question 1 and your answer to question 4 to find Ivan's position when he glances up. That position is... at the east portal of the tunnel, just as happens in the train's frame. Relativity makes sense, it just doesn't make common sense.