Model Solutions for Short Answer Portion of Sample Final Exam — Einstein and Relativity

Veronica races down a 200-foot-long straight track at uniform speed $\frac{4}{5}c$ (so that $\sqrt{1-(V/c)^2} = \frac{3}{5}$). There are clocks at the start line and at the finish line. The start-line clock reads 0 nans as Veronica passes.

1. *How much time does her race require (in earth's frame)? [This is the time that will be displayed on the finish-line clock when Veronica passes it.]*

This problem uses the definition

$$\text{speed} = \frac{\text{distance traveled}}{\text{time elapsed}}.$$

Applied to our case,

$$\text{time elapsed} = \frac{\text{distance traveled}}{\text{speed}} = \frac{200 \text{ ft}}{\frac{4}{5} \text{ ft/nan}} = \tfrac{5}{4}(200 \text{ nan}) = 250 \text{ nan}.$$

2. *How much time does Veronica's wristwatch tick off during her race?*

This is a time dilation problem. The moving wristwatch ticks off the smaller time $\frac{3}{5}(250 \text{ nan}) = 150$ nan. [If you wish to use formulas, set the time elapsed T to 250 nan, and solve for the time T_0 ticked off by the moving clock.]

3. *In Veronica's frame, the finish line moves toward her. How long is the racetrack in Veronica's frame?*

This is a length contraction problem. The racetrack has length $L_0 = 200$ ft in its own frame, so it has a shorter length $L = \frac{3}{5}(200 \text{ ft}) = 120$ ft in Veronica's frame.

4. *In Veronica's frame, when she passes the start line (and the start-line clock reads 0 nans), what does the finish-line clock read?*

This is a relativity of synchronization problem. The finish-line clock is the rear clock in Veronica's frame, so it is set ahead by

$$\frac{L_0 V}{c^2} = \frac{(200 \text{ ft})(\frac{4}{5}c)}{c^2} = \frac{(200 \text{ ft})(\frac{4}{5})}{c} = \frac{(200 \text{ ft})(\frac{4}{5})}{1 \text{ ft/nan}} = \tfrac{4}{5}(200) \text{ nan} = 160 \text{ nan}.$$

5. *In Veronica's frame, how much time does the finish-line clock tick off during her race?*

Another time dilation problem. The answer to question 2 tells us that in Veronica's frame, the time elapsed is 150 nans. The moving finish-line clock ticks off a smaller time $\frac{3}{5}(150 \text{ nan}) = 90$ nan.

6. *Sum the answers to 4 and 5, and compare with the result of 1.*

In Veronica's frame, the finish-line clock starts out set at 160 nans, and ticks off 90 nans during her race, so at the end of the race, when Veronica passes the finish line, that clock reads $160 \text{ nan} + 90 \text{ nan} = 250$ nan. This agrees exactly with the result already determined through reasoning in earth's frame at question 1.

Time and again I am impressed with the remarkable consistency of "this strange and beautiful universe, our home." (Quoted from Charles Misner, Kip Thorne, and John Archibald Wheeler, *Gravitation*, page v.)