Joust

Sir Lancelot and Sir Mordred each has a lance of rest length 17 feet. During a joust, each gallops toward the other at speed $\frac{3}{5}c$ relative to the Earth.

Earth's frame: horses resting in position before the joust

Lancelot   Mordred
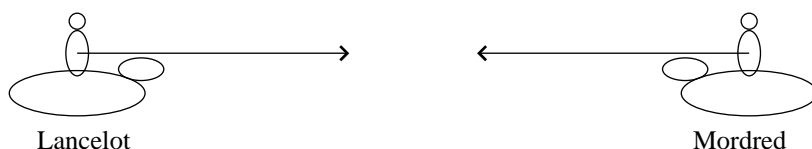
(a) The crowd in the stands sees a fair fight, because each knight has a lance of equal length. How long is each lance in the Earth's frame?

When the horses are running, each lance has length

$$L = \sqrt{1 - (V/c)^2}\, L_0 = \tfrac{4}{5}(17 \text{ feet}) = 13.6 \text{ feet}.$$

Earth's frame: start of joust

Lancelot   Mordred

(b1) How fast is Mordred moving in Lancelot's frame?

Einstein's speed addition formula: If

w_b represents the speed of Mordred relative to Lancelot,
v_b represents the speed of Mordred relative to the Earth $(-\frac{3}{5}c)$, and
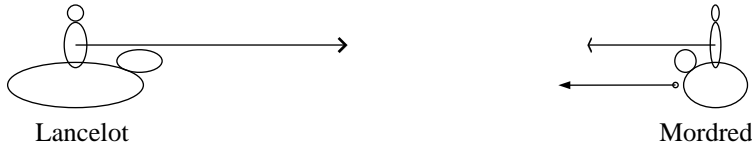V represents the speed of the Earth relative to Lancelot $(-\frac{3}{5}c)$,

then

$$
\begin{aligned}
w_b &= \frac{v_b + V}{1 + v_b V/c^2} \\
&= \frac{-\frac{3}{5}c - \frac{3}{5}c}{1 + \frac{3}{5}\frac{3}{5}} \\
&= -\frac{15}{17}c.
\end{aligned}
$$

(b2) How long is Mordred's lance in Lancelot's frame?

$$L = \sqrt{1 - (w_b/c)^2}\, L_0 = \tfrac{8}{17}(17 \text{ feet}) = 8 \text{ feet.}$$

Lancelot's frame: start of joust
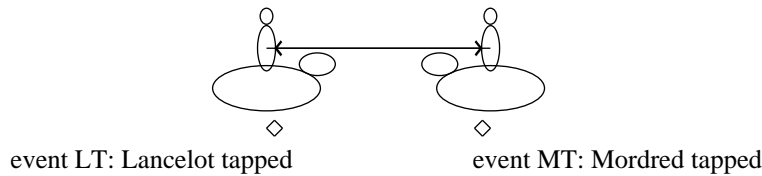

Lancelot   Mordred

(c) How fast is Lancelot moving in Mordred's frame? How long is Lancelot's lance in Mordred's frame?

In Lancelot's frame, his own lance is much longer than his opponent's, so Lancelot expects to win easily. However, the same holds for Mordred! Meanwhile, in the crowd's frame, the two lances are equally long.

In the crowd's frame, each knight is tapped by the opposing knight's lance simultaneously. Also, both knights are unhorsed simultaneously.

Earth's frame: lances tap breastplates simultaneously

event LT: Lancelot tapped   event MT: Mordred tapped

(d1) Which knight is tapped first in Lancelot's frame?

Lancelot's frame moves right relative to Earth's frame, so the two events move left in Lancelot's frame, and the rear event (Mordred tapped) happens first:

Lancelot's frame: those two events are not simultaneous

event LT: Lancelot tapped   event MT: Mordred tapped
                                         rear event happens first
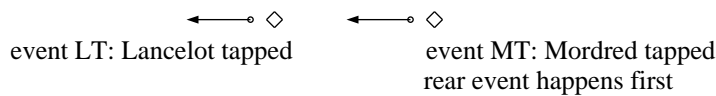
(d2) Which knight is tapped first in Mordred's frame?

(e) Is Lancelot unhorsed at the instant that Mordred's lance taps his breastplate?

(f) In each knight's frame, one knight is unhorsed by another knight who has already been struck. How can a struck knight succeed in unhorsing his opponent?

There are four events:

- event MT: Mordred tapped by tip of Lancelot's lance, which starts to compress

- event LT: Lancelot tapped by tip of Mordred's lance, which starts to compress

- event MU: Mordred is unhorsed

- event LU: Lancelot is unhorsed

In the Earth's frame the first two events are simultaneous, and the last two events are simultaneous. In Lancelot's frame, the four events happen in the sequence presented in the list above.

(g) In what sequence do these four events happen in Mordred's frame?