Two-electron ions

Let Z represent the variational parameter in [7.27].
Z_N represent the nuclear charge (1 for H^-, 3 for Li^+).

We follow the argument of Griffiths pages 302–303, *except*:

- In equations [7.28] (twice) and [7.29], change $(Z-2)$ to $(Z-Z_N)$.
- Equation [7.32] becomes

$$\langle H\rangle = [2Z^2 - 4Z(Z-Z_N) - \tfrac{5}{4}Z]E_1 = [-2Z^2 + (4Z_N - \tfrac{5}{4})Z]E_1.$$

- In equations [7.32$\frac{1}{2}$] and [7.33], change 27 to $(16Z_N - 5)$.

This changes the equation answers as follows:

Equation [7.33] becomes

$$Z = \frac{16Z_N - 5}{16} = Z_N - \tfrac{5}{16}.$$

Equation [7.34] becomes

$$\langle H\rangle = [-2Z^2 + (4Z_N - \tfrac{5}{4})Z]E_1 = \frac{(16Z_N - 5)^2}{2^7}E_1.$$

And (remembering $E_1 = -13.6$ eV) it changes the numerical ground state energy estimates to:

$$\text{For H}^-,\ Z_N = 1 \quad \text{so} \quad \frac{11^2}{2^7}E_1 = \frac{121}{128}E_1 = -12.9 \text{ eV}.$$

$$\text{For Li}^+,\ Z_N = 3 \quad \text{so} \quad \frac{(43)^2}{2^7}E_1 = \frac{1849}{128}E_1 = -196 \text{ eV}.$$