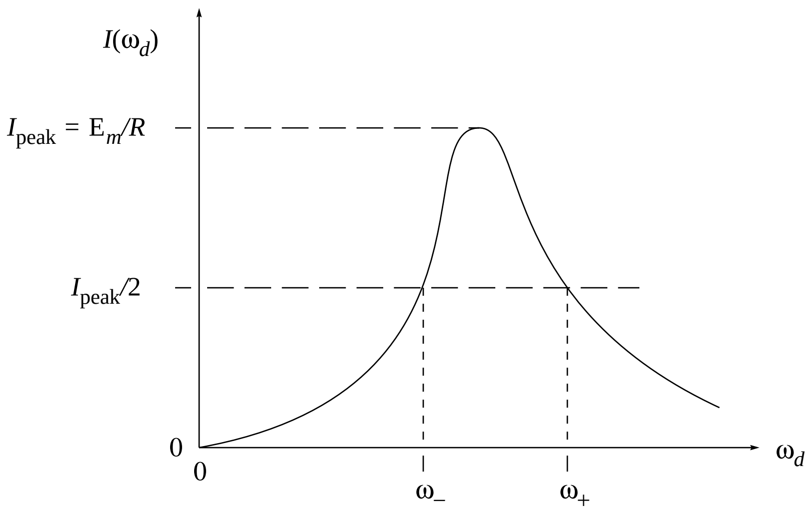
Width of a resonance curve

The resonance curve is given by LSM equation 15.15, namely (where I have used the notation from class, not from LSM)

$$I(\omega_d) = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}. \tag{1}$$



Objective: Find the two values, ω_- and ω_+, at which $I(\omega_d) = \mathcal{E}_m/(2R)$.

$$\begin{aligned}
\frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_\pm L - 1/\omega_\pm C)^2}} &= \frac{\mathcal{E}_m}{2R} && (2)\\
R^2 + (\omega_\pm L - 1/\omega_\pm C)^2 &= 4R^2 && (3)\\
(\omega_\pm L - 1/\omega_\pm C)^2 &= 3R^2 && (4)\\
\omega_\pm L - 1/\omega_\pm C &= \pm\sqrt{3R^2} \qquad [\![\text{Notice the } \pm \text{ out front!}]\!] && (5)\\
\omega_\pm^2 L \mp \sqrt{3R^2}\,\omega_\pm - 1/C &= 0 && (6)
\end{aligned}$$

Solve using quadratic formula

$$\omega_\pm = \frac{\pm\sqrt{3R^2} \pm \sqrt{3R^2 + 4L/C}}{2L}. \tag{7}$$

This gives us four values for $\omega_\pm$! Which two of these are physically relevant? Because

$$\sqrt{3R^2 + 4L/C} > \sqrt{3R^2},$$

the two roots with $-\sqrt{3R^2 + 4L/C}$ are negative and hence physically irrelevant. Thus

$$\omega_- = \frac{1}{2L}\left(\sqrt{3R^2 + 4L/C} - \sqrt{3R^2}\right); \qquad \omega_+ = \frac{1}{2L}\left(\sqrt{3R^2 + 4L/C} + \sqrt{3R^2}\right). \tag{8}$$

and

$$\Delta\omega = \omega_+ - \omega_- = \frac{\sqrt{3}\,R}{L}. \tag{9}$$

Remarkably, this is independent of C!

Grading: Two points for reaching each of these five milestones: equation (1), equation (2), equation (6), equation (8), equation (9).