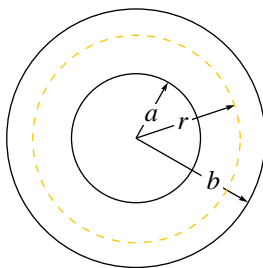


## The thick pipe of current

(a.) *Qualitative considerations:* By symmetry, the magnitude of  $\vec{B}$  can depend only upon the distance  $r$  from the cylinder axis. In addition, because  $\vec{B}$  is the sum of many circular contributions all in the plane perpendicular to the cylinder axis,  $\vec{B}$  must be in this plane. Finally,  $\vec{B}$  must be tangential to circles around the axis because any radial component would lead to

$$\oint_{\text{surface}} \vec{B} \cdot \hat{n} dA \neq 0.$$

In summary, lines of  $\vec{B}$  must be circles centered on the cylinder axis.



*Quantitative considerations:* For any circle centered on the axis,

$$\vec{B} \cdot d\vec{\ell} = B(r) d\ell \quad \text{and} \quad \oint \vec{B} \cdot d\vec{\ell} = B(r) \oint d\ell = B(r) 2\pi r$$

so, from Ampere's law,

$$B(r) = \frac{\mu_0 I_{\text{linked}}(r)}{2\pi r}.$$

Meanwhile:

For  $r < a$ ,  $I_{\text{linked}}(r) = 0$ .

For  $r > b$ ,  $I_{\text{linked}}(r) = i$ .

For  $a < r < b$ ,  $\frac{I_{\text{linked}}(r)}{i} = \frac{\text{area between } a \text{ and } r}{\text{area between } a \text{ and } b} = \frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}$ , whence  $I_{\text{linked}}(r) = i \frac{(r^2 - a^2)}{(b^2 - a^2)}$ .

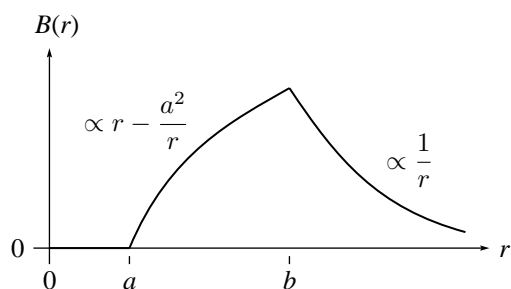
Thus

$$B(r) = \begin{cases} 0 & r < a \\ \frac{\mu_0 i}{2\pi r} \frac{(r^2 - a^2)}{(b^2 - a^2)} & a < r < b \\ \frac{\mu_0 i}{2\pi r} & b < r \end{cases}$$

(b.)  $B(r)$  is continuous at  $r = a$  and  $r = b$ :  $B(a) = 0$  and  $B(b)$  is appropriate for the  $\vec{B}$  of a long thin wire, as expected.

If  $a = 0$  this is the situation of LSM example 12.7 on page 536 (changing our  $b$  to LSM's  $a$ ). And sure enough, if you plug  $a = 0$ , and change  $b$  to  $a$  into the equation above you come up with the equation at the bottom of page 536.

(c.)



The graphs of the left-most and right-most parts of the function are straightforward. For the middle portion ( $a < r < b$ ) note that the slope is

$$\frac{dB}{dr} = \frac{\mu_0 i}{2\pi(b^2 - a^2)} \left( 1 + 2\frac{a^2}{r^2} \right)$$

so that (i) the slope is always positive — never zero or negative — and (ii) as  $r$  increases, the slope decreases.