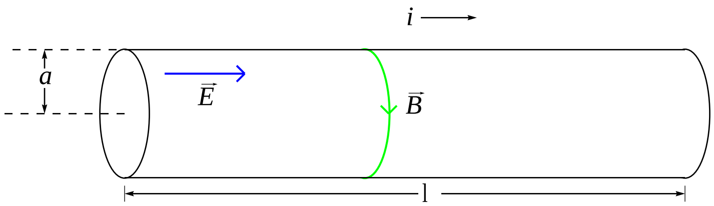
How does i^2R heating power get to a resistor?



(a.) To find $\vec{S} = (1/\mu_0)\vec{E} \times \vec{B}$ at the surface we first find $\vec{E}$ and $\vec{B}$ at the surface. The electric field everywhere in the cylinder (including at the surface) comes through the microscopic form of Ohm's law:

$$\vec{E} = \rho\vec{J} \quad \text{so} \quad |\vec{E}| = \rho\frac{i}{\pi a^2}.$$

And of course the magnetic field at the surface is that of a long straight wire, namely

$$|\vec{B}| = \frac{\mu_0 i}{2\pi a}.$$

The directions of $\vec{E}$ and $\vec{B}$ are shown in the figure. At the surface, they are perpendicular and $\vec{E} \times \vec{B}$ points inward. The magnitude of $\vec{S}$ is

$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E} \times \vec{B}| = \frac{1}{\mu_0}|\vec{E}|\,|\vec{B}| = \frac{\rho i^2}{2\pi^2 a^3}.$$

(b.) The rate at which electromagnetic energy flows into the resistor is

$$-\oint_{\text{surface}} \vec{S}\cdot\hat{n}\,dA = |\vec{S}|(\text{side area}) = |\vec{S}|(2\pi a\ell) = \frac{\rho i^2 \ell}{\pi a^2}.$$

(c.) The resistance of the cylinder is

$$R = \rho\frac{\ell}{(\text{cross-sectional area})} = \rho\frac{\ell}{\pi a^2}$$

so

$$\text{rate of EM energy flow into resistor} = i^2 R.$$

Grading: 2 points for starting off (e.g. a diagram); then one point each for (1) writing expression for $\vec{S}$; (2) expression for $|\vec{E}|$; (3) expression for $|\vec{B}|$; (4) direction of $\vec{S}$; (5) expression for $|\vec{S}|$; (6) answer to part **b**; (7) expression for R; (8) expression for "rate of EM energy flow".