Experimental detection of the Maxwell term

(a.) If A represents the area of each plate, and d the distance between plates, then the capacitance is $C = \epsilon_0(A/d)$. Remembering also that $\Delta V = Ed$, we have

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d(EA)}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d(\Delta V/d)}{dt} = \epsilon_0 (A/d) \frac{d(\Delta V)}{dt} = C \frac{d(\Delta V)}{dt}.$$

(b.) According to part (a.),

$$i_d = C \frac{dV(t)}{dt}.$$

But

$$V(t) = V_m \sin(\omega t) \quad \text{so} \quad \frac{dV(t)}{dt} = V_m \omega \cos(\omega t),$$

so

$$i_d = C V_m \omega \cos(\omega t).$$

Thus the amplitude of the displacement current is

$$C V_m \omega = C V_m 2\pi f = (100 \text{ pF})(174 \text{ kV}) 2\pi (50.0 \text{ Hz}) = 5.47 \text{ mA}.$$

(c.) To make a large displacement current, one needs either a large C, a large V_m, or a large f (fast changes). Given the experimental constraints in place in 1929, van Cauwenberghe chose to use a large V_m.

Grading: 3 points for part (a); 3 points for equation in part (b); 3 points for number in part (b); 1 point for part (c).